

Physics 331 Problem 2

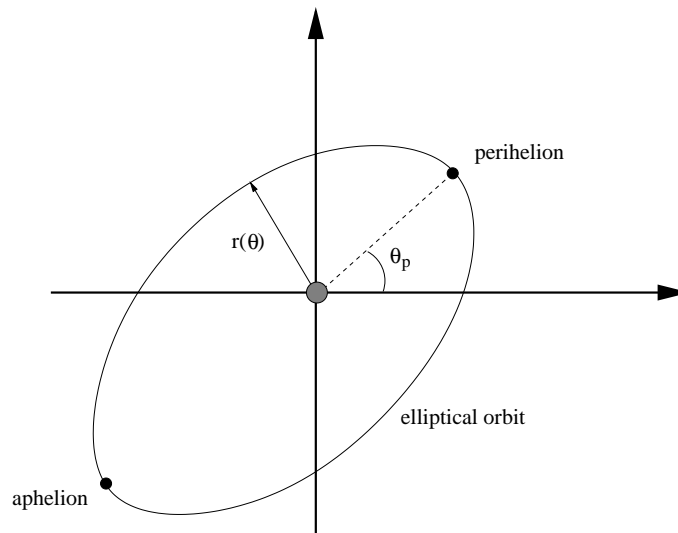
Due Thursday, Sept. 30, 2004

Elliptical Orbits

Kepler's first law states that a planet (or spacecraft) will follow an elliptical orbit about the sun.¹ In polar coordinates an elliptical orbit is given by the curve:

$$r(\theta) = \frac{a(1 - \epsilon^2)}{1 + \epsilon \cos(\theta - \theta_P)}. \quad (1)$$

The three parameters of the orbit are ϵ the eccentricity, a the semi-major axis, and θ_P the angle of perihelion (the point closest the Sun).



The above sketch shows an elliptical orbit with perihelion at angle θ_P . Note that ϵ describes how “non-circular” the orbit is. Setting $\epsilon = 0$ gives the curve $r(\theta) = a$, a perfect circle. The orbital parameters for Mars are:

$$\begin{aligned} a &= 2.279 \times 10^8 \text{ km} \\ \epsilon &= 0.093 \\ \theta_P &= 234^\circ \end{aligned}$$

¹This assumes the orbiting body is affected only by the gravitational pull of the Sun. We will relax this assumption in the future.

The problem:

A spacecraft A is launched from Earth into an elliptical orbit $r_A(\theta)$ with the parameters

$$\begin{aligned}a &= 1.961 \times 10^8 \text{km} \\ \epsilon &= 0.25 \\ \theta_P &= 0^\circ\end{aligned}$$

The question you must answer is: **At which angle(s) θ does the orbit of the spacecraft cross the orbit of Mars?** HINT: At the crossing point $r_A(\theta) = r_{mars}(\theta)$, each of the curves is given by equation (1) with the appropriate orbital parameters.

Please e-mail the following to `kankel@solar.physics.montana.edu`:

1. The angle or angles at which the orbits cross.
2. A copy of the program you wrote.
3. A copy of the command(s) used to run your program.