

The Journey to Mars

We have planned a journey to Mars along the following scenario. The mission will be assembled in geosynchronous Earth orbit and depart at local midnight (i.e. when the spacecraft is on the opposite side of the Earth from the Sun). This means the journey itself begins at a position

$$\mathbf{x}_0 = (R_e + \rho) \times [\cos(\phi_e) \hat{\mathbf{x}} + \sin(\phi_e) \hat{\mathbf{y}}] ,$$

where $\rho = 4.22 \times 10^7$ meters is the radius of geosynchronous orbit and ϕ_e is the angle of Earth in our solar-system coordinates on departure day d_* . The spacecraft is at \mathbf{x}_0 after firing its engines and has a velocity (in the inertial reference frame) of

$$\mathbf{v}_0 = v_0 \times [-\sin(\phi_e) \hat{\mathbf{x}} + \cos(\phi_e) \hat{\mathbf{y}}] ,$$

where $v_0 = 3.60 \times 10^4$ m/sec.

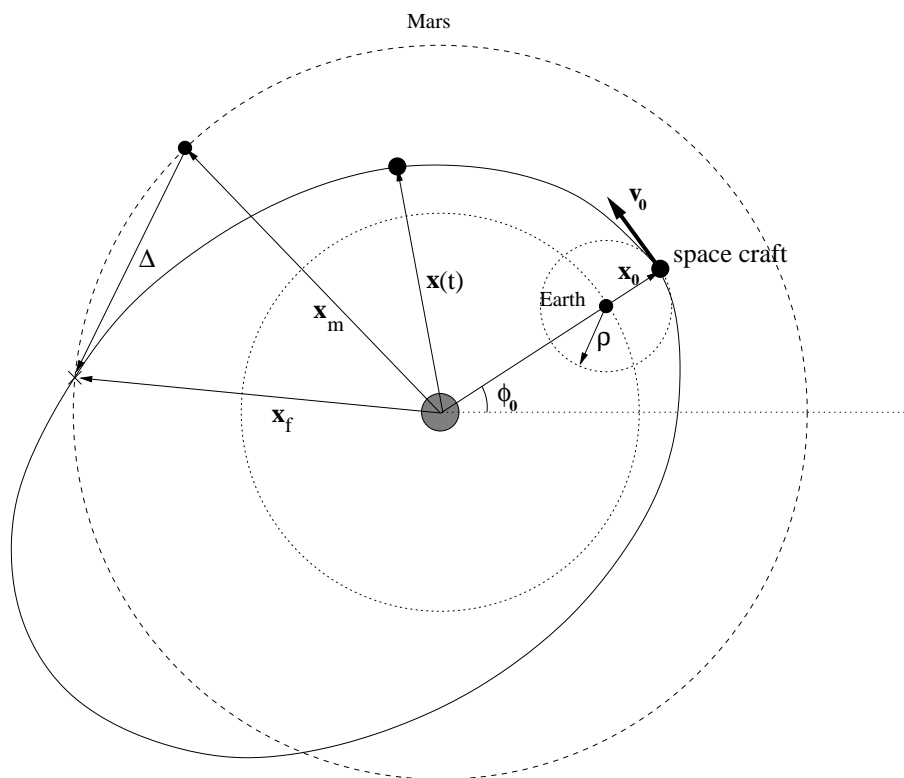


Figure 1: Plan for the Martian Journey

The departure date will be denoted by day d_* numbered from Jan 1, 2000 (even though dates under consideration are in 2002). You must write a program which calculates the

trajectory of the spacecraft from $\mathbf{x}(t=0) = \mathbf{x}_0$ beginning on an arbitrary departure date until it crosses the orbit of Mars:

$$|\mathbf{x}_f| = R_m = 2.279 \times 10^{11} \text{ m} .$$

You wish to know 1. on which day does this occur, and 2. how close is Mars,

$$\Delta = |\mathbf{x}_f - \mathbf{x}_m(t_f)| .$$

Your program should include the gravitational pulls of the Sun, Earth and Mars. In other words it should solve the differential equation

$$\ddot{\mathbf{x}} = -GM_s \frac{\mathbf{x}}{|\mathbf{x}|^3} + GM_e \frac{\mathbf{x}_e(t) - \mathbf{x}}{|\mathbf{x}_e(t) - \mathbf{x}|^3} + GM_m \frac{\mathbf{x}_m(t) - \mathbf{x}}{|\mathbf{x}_m(t) - \mathbf{x}|^3}$$

where $\mathbf{x}_e(t)$ and $\mathbf{x}_m(t)$ are the positions of Earth and Mars at t seconds. Note that $t = 86,400 \times d$ where d is the day-number, since there are 86,400 seconds in a day.

The calculation should be performed for the following tentative departure dates:

- a. March 15, 2002, $d_* = 805$
- b. April 10, 2002, $d_* = 831$
- c. April 30, 2002, $d_* = 851$

Please e-mail the following to my address `kankel@mithra.physics.montana.edu`:

1. The durations for each flight (in days) and distance of approach Δ .
2. A copy of the program(s) you wrote.
3. A copy of the command(s) used to run your program, which should let me know what size time-step you have taken, and which solution algorithm you used.
4. A comment on the reliability of 1.

| i | Mass M_i (kg) | Orbital radius R_i (m) | Orbital Period P_i (days) |
|-------|-----------------------|--------------------------|-----------------------------|
| Sun | 1.99×10^{30} | — | — |
| Earth | 5.94×10^{24} | 1.496×10^{11} | 365.256 |
| Mars | 6.42×10^{23} | 2.279×10^{11} | 686.980 |

Table 1: Some useful quantities