

PHYSICS 423, E&M I

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4 MOST IMPORTANT UNDERGRAD PHYSICS COURSES:

- CLASSICAL MECHANICS
- E & M
- QUANTUM MECHANICS
- STATISTICAL & THERMAL PHYSICS

IF YOU DO WELL IN THESE COURSES, IT MAKES A BIG IMPRESSION ON GRAD SCHOOL ADMISSIONS COMMITTEES.

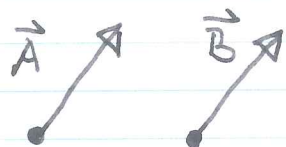
COURSE WEBSITE: solar.physics.montana.edu/kankel/ph423/

- REVIEW SYLLABUS
- HOMEWORK
- CLASS PHOTO?

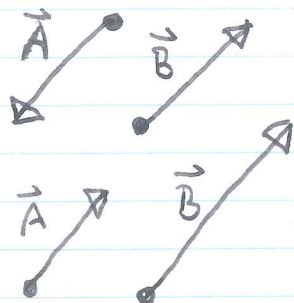
WE START TODAY WITH VECTOR ALGEBRA, GRIFFITHS § 1.1

GRIFFITHS § 1.1 - VECTOR ALGEBRA

A VECTOR (\vec{A}) HAS BOTH A MAGNITUDE AND A DIRECTION. $(A = |\vec{A}|, \text{LENGTH})$



$(\vec{A} = \vec{B})$ — IT DOESN'T MATTER WHERE I DRAW THESE 2 VECTORS, SO LONG AS MAGNITUDE (LENGTH) & DIRECTION MATCH ("PARALLEL TRANSPORT")



$(\vec{A} \neq \vec{B}, \vec{A} = -\vec{B})$

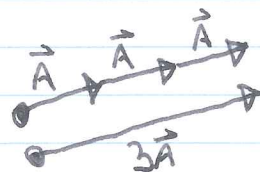
$(\vec{A} \neq \vec{B}, \vec{B} = 2\vec{A})$

SCALAR MULTIPLICATION

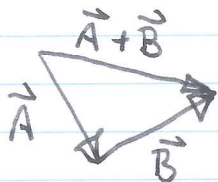
Q: WHAT DO WE MEAN BY $3\vec{A}$?

A: $3\vec{A} = \vec{A} + \vec{A} + \vec{A}$

$|\vec{3A}| = 3|\vec{A}| = 3A$



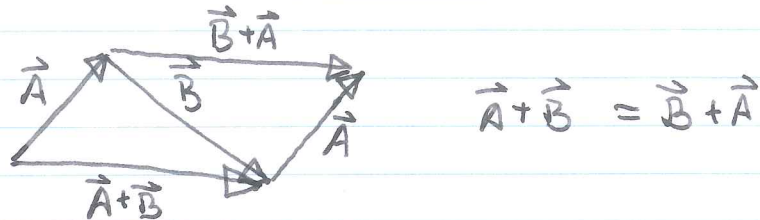
THE ABOVE IS AN EXAMPLE OF VECTOR ADDITION:



} ADDITION IS DONE HEAD-TO-TAIL.

SCALAR MULTIPLICATION CAN BE USED TO CONSTRUCT UNIT VECTORS:

$\hat{A} \equiv \frac{1}{A} \vec{A}$
 $|\hat{A}| = \left| \frac{\vec{A}}{A} \right| = \frac{|\vec{A}|}{A} = \frac{A}{A} = 1$

PROPERTIES:VECTOR ADDITION IS COMMUTATIVE:VECTOR ADDITION IS ASSOCIATIVE:

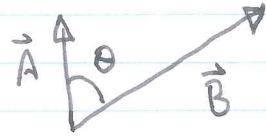
$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

DISTRIBUTIVE PROPERTY OF SCALAR MULTIPLICATION OVER VECTOR ADDITION:

$$a(\vec{A} + \vec{B}) = a\vec{A} + a\vec{B}$$

DOT PRODUCT

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad \leftarrow \text{SCALAR RESULT!}$$



RESULT INCREASES AS $\theta \rightarrow 0$.
RESULT SHRINKS AS $\theta \rightarrow \pi$.

NOTE HOW I HAVE DRAWN THESE TAIL-TO-TAIL TO MEASURE θ .

COMMUTATIVE: $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ (WHY?)

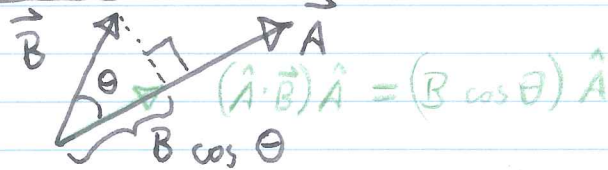
Q: ASSOCIATIVE? (DISCUSS) $\vec{A} \cdot (\vec{B} \cdot \vec{C}) = (\vec{A} \cdot \vec{B}) \cdot \vec{C}$??

EXERCISE GIVEN 2 VECTORS $\vec{A}, \vec{B} \dots$

Q: WHAT IS THE INTERPRETATION OF $(\hat{A} \cdot \vec{B}) \hat{A}$?

$$\hat{A} \cdot \vec{B} = \frac{\vec{A}}{A} \cdot \vec{B} = \frac{AB \cos \theta}{A} = B \cos \theta$$

A: PROJECTION OF \vec{B} ONTO \vec{A} .



"HOW MUCH OF \vec{B} IS IN THE \vec{A} -DIRECTION?"

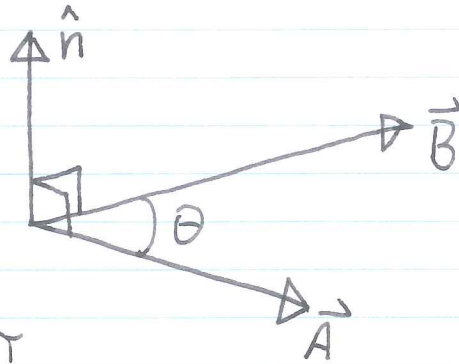
Q: WHAT DOES $\vec{A} \cdot \vec{B} = 0$ MEAN?

A: $\vec{A} \perp \vec{B}$

(5)

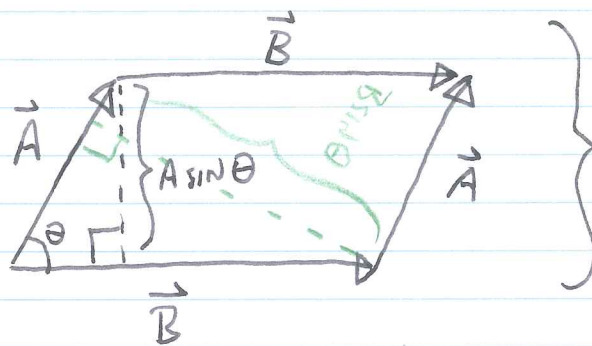
CROSS PRODUCT $\vec{A} \times \vec{B} = \hat{n} AB \sin \theta$

LET \hat{n} BE PERPENDICULAR
TO THE PLANE DEFINED
BY \vec{A} & \vec{B} , I.E.
 $\hat{n} \cdot \vec{A} = \hat{n} \cdot \vec{B} = 0$.



THE DIRECTION OF \hat{n}
IS GIVEN BY THE RIGHT
HAND RULE.

$$|\vec{A} \times \vec{B}| = AB \sin \theta ; \text{ INTERPRETATION:}$$



$$|\vec{A} \times \vec{B}| = \text{AREA OF PARALLELOGRAM}$$

Q: DOES $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ (COMMUTATIVE)?

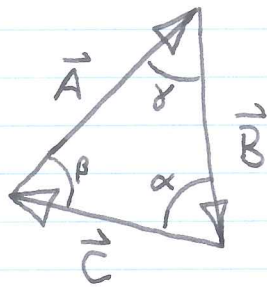
A: NO. $|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}|$, BUT
 $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

NOTE THAT : $\vec{A} \cdot \vec{A} = A^2$;
 $\vec{A} \times \vec{A} = \vec{0}$.

DISTRIBUTIVE PROPERTY OF X OVER + :

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}.$$

(6)

EXAMPLE - DERIVATION OF THE LAW OF SINESASIDE

Q: HOW DOES γ RELATE TO THE ANGLE θ , AS IN $\vec{A} \cdot \vec{B} = AB \cos \theta$, AND $|\vec{A} \times \vec{B}| = AB \sin \theta$?

A: $\gamma = \pi - \theta$.

$$\vec{A} + \vec{B} + \vec{C} = \vec{0} \quad (\text{DISCUSS})$$

$$|\vec{A} \times \vec{B}| = AB \sin \gamma$$

$$= |\vec{A} \times (-\vec{A} - \vec{C})| = |\vec{C} \times \vec{A}| = AC \sin \beta$$

$$= |(-\vec{B} - \vec{C}) \times \vec{B}| = |\vec{B} \times \vec{C}| = BC \sin \alpha$$

$$\therefore AB \sin \gamma = AC \sin \beta = BC \sin \alpha$$

DIVIDE BY ABC:

$$\boxed{\frac{\sin \gamma}{C} = \frac{\sin \beta}{B} = \frac{\sin \alpha}{A}} \quad \text{LAW OF SINES}$$

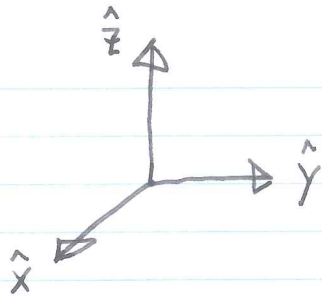
EXERCISE: USE THE SAME VECTOR TRIANGLE TO PROVE THE LAW OF COSINES,

$$C^2 = A^2 + B^2 - 2AB \cos \gamma.$$

HINT: EVALUATE $\vec{C} \cdot \vec{C}$ IN TERMS OF \vec{A} AND \vec{B} .

COMPONENT FORM

ALWAYS USE A RIGHT-HANDED COORDINATE SYSTEM!



WE WISH TO WRITE A VECTOR $\vec{A} \in \mathbb{R}^3$ IN TERMS OF 3 COMPONENTS:

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} .$$

(THUS, $\vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2$ — PYTHAGOREAN THEOREM)

THE COMPONENTS A_x, A_y, A_z ARE UNIQUELY GIVEN BY THESE EXPRESSIONS:

$$A_x = \vec{A} \cdot \hat{x} , \quad A_y = \vec{A} \cdot \hat{y} , \quad A_z = \vec{A} \cdot \hat{z} \quad \star$$

THUS, \vec{A} IS THE SUM OF PROJECTIONS ALONG ALL THREE COORDINATES:

$$\vec{A} = (\vec{A} \cdot \hat{x}) \hat{x} + (\vec{A} \cdot \hat{y}) \hat{y} + (\vec{A} \cdot \hat{z}) \hat{z} .$$

PROOF OF \star :

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

DOT BOTH SIDES WITH \hat{x} :

$$\begin{aligned} \vec{A} \cdot \hat{x} &= A_x (\hat{x} \cdot \hat{x}) + A_y (\hat{y} \cdot \hat{x}) + A_z (\hat{z} \cdot \hat{x}) \\ &= A_x \checkmark \end{aligned}$$

SIMILARLY FOR \hat{y}, \hat{z} .

CROSS PRODUCT IN COMPONENT FORM

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} ; \quad \vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z} .$$

$$\vec{A} \times \vec{B} = [A_x \hat{x} + A_y \hat{y} + A_z \hat{z}] \times [B_x \hat{x} + B_y \hat{y} + B_z \hat{z}]$$

Now, $\hat{x} \times \hat{x} = \hat{y} \times \hat{y} = \hat{z} \times \hat{z} = 0, *$

$$\hat{x} \times \hat{y} = -(\hat{y} \times \hat{x}) = \hat{z},$$

$$\hat{y} \times \hat{z} = -(\hat{z} \times \hat{y}) = \hat{x},$$

$$\hat{z} \times \hat{x} = -(\hat{x} \times \hat{z}) = \hat{y}.$$

USING THE DISTRIBUTIVE PROPERTY,

$$\vec{A} \times \vec{B} = \begin{matrix} A_x B_y \hat{z} & - & A_x B_z \hat{y} \\ -A_y B_x \hat{z} & + & A_y B_z \hat{x} \\ +A_z B_x \hat{y} & - & A_z B_y \hat{x} \end{matrix}$$

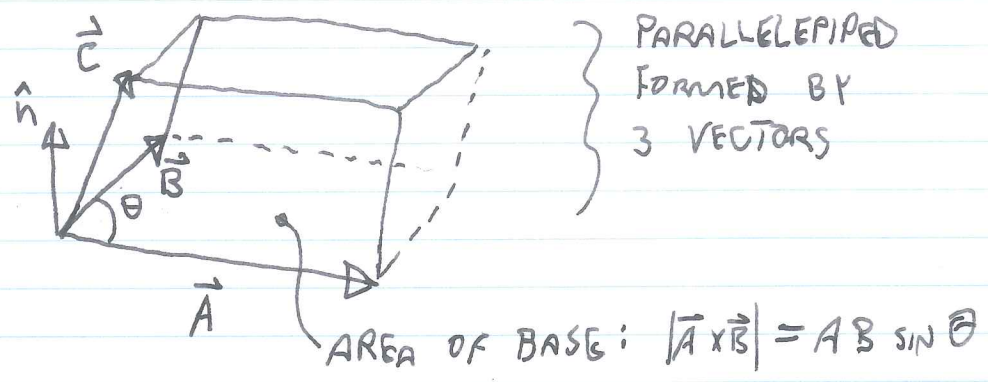
DETERMINANT NOTATION:

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

*I WILL OFTEN WRITE THE ZERO VECTOR, $\vec{0}$, LIKE SCALAR ZERO, SINCE $\vec{0}$ HAS NO DIRECTION.

TRIPLE PRODUCTS (§ 1.1.3)

A GEOMETRICAL INTERPRETATION OF $\vec{c} \cdot (\vec{a} \times \vec{b})$:



$$\text{VOLUME } V = \vec{c} \cdot (\vec{a} \times \vec{b})$$

$$= (\vec{c} \cdot \hat{n}) (AB \sin \theta)$$

NOTE HOW $\vec{a} \times \vec{b}$ RATHER THAN $\vec{b} \times \vec{a}$ WAS CHOSEN TO GET $V > 0$.

OF COURSE, ANY 2 VECTORS COULD BE CHOSEN TO DESIGNATE THE "BASE", SO

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

"SCALAR TRIPLE PRODUCT"

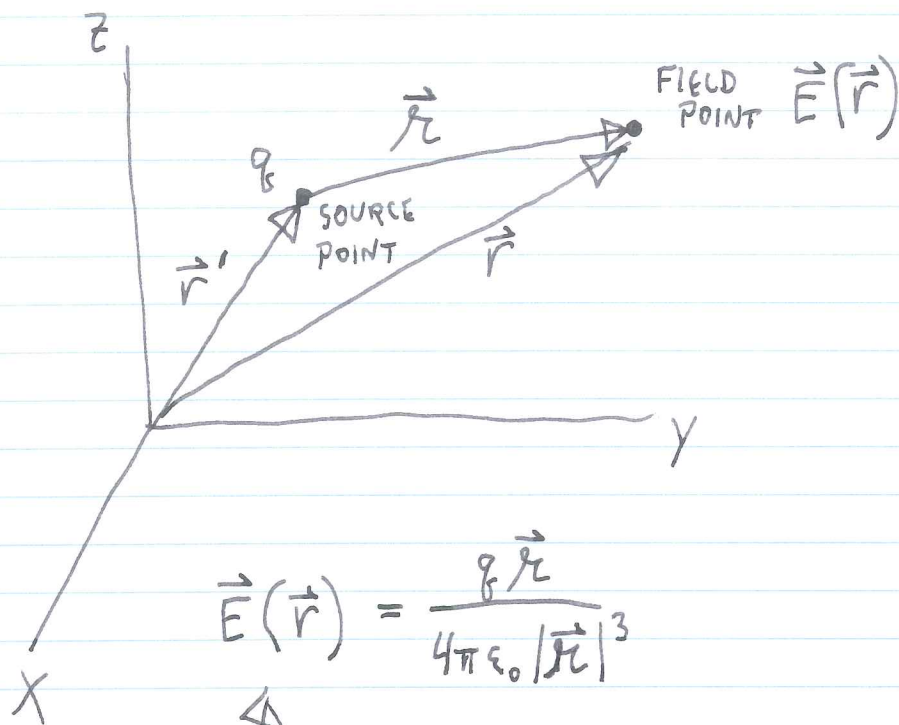
THE OTHER TRIPLE PRODUCT, WHICH IS VECTOR-VALUED, OBEYS THE "BAC-CAB RULE":

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$

THIS CAN BE PROVEN BY WRITING IT OUT CAREFULLY IN COMPONENT FORM (PROB 1.5), THOUGH IT'S A BIT TEDIOUS.

THESE AND OTHER FORMULAE APPEAR IN THE END PAPERS OF GRIFFITHS. I WILL NOT REQUIRE YOU TO MEMORIZE THEM, BUT IT'S HELPFUL TO BE FAMILIAR AND COMFORTABLE WITH USING THEM.

NOTATION IN GRIFFITHS:



ELECTROMAGNETIC THEORY IS FULL OF VECTOR
FIELDS ($\vec{E}(\vec{r})$, $\vec{B}(\vec{r})$, AND OTHERS).

THAT'S WHY WE ARE SPENDING A WHOLE
 CHAPTER ON VECTOR ANALYSIS.

IN COMING SECTIONS, WE WILL EXPLORE
 THE DIFFERENTIAL AND INTEGRAL CALCULUS
 OF VECTOR FIELDS.

- READ § 1.2
- START THE HOMEWORK!