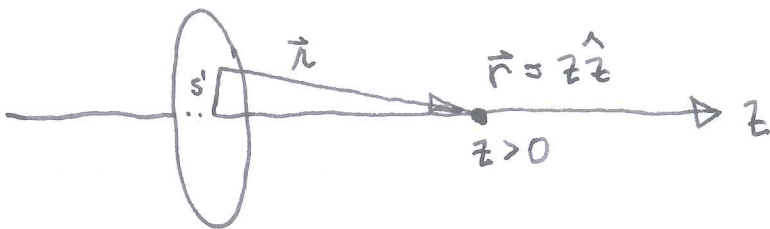


Let's return to this example and consider the case $z < 0$.

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EXAMPLE: UNIFORMLY CHARGED DISK



$$\begin{aligned} \vec{r}' &= s' \hat{s}' \\ \vec{r} &= z \hat{z} \\ \vec{r} &= z \hat{z} - s' \hat{s}' \\ r &= [z^2 + s'^2]^{1/2} \end{aligned}$$

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma \vec{r}}{r^2} da = \frac{\sigma}{4\pi\epsilon_0} \int_{s'=0}^R \int_{\phi=0}^{2\pi} \frac{(z \hat{z} - s' \hat{s}') s' d\phi' ds'}{(z^2 + s'^2)^{3/2}}$$

The constraint $z > 0$ was employed in taking the limits after integration:

$$= \frac{\sigma}{2\epsilon_0} \int_{s'=0}^R \frac{z \hat{z} s' ds'}{(z^2 + s'^2)^{3/2}} = \frac{\sigma}{2\epsilon_0} \frac{-z \hat{z}}{(z^2 + s'^2)^{1/2}} \Big|_{s'=0}^R$$

The square root of z^2 is not z , but $|z|$.

$$= -\frac{\sigma z \hat{z}}{2\epsilon_0} \left[\frac{1}{\sqrt{z^2 + R^2}} - \frac{1}{|z|} \right] = \frac{\sigma \hat{z}}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + R^2}} \right] \text{sgn}(z)$$

The "signum" of z is defined as $\text{sgn}(z) = z/|z|$.

IF WE HOLD FIXED THE TOTAL CHARGE $q = \pi R^2 \sigma$, AND TAKE THE LIMIT $R \rightarrow 0$,

$$\vec{E} = \frac{q \hat{z}}{2\pi \epsilon_0} \left[\frac{1}{R^2} - \frac{z}{R^2 \sqrt{z^2 + R^2}} \right] \text{sgn}(z)$$

BINOMIAL EXPANSION $(1 + \epsilon)^x = 1 + x\epsilon + O(\epsilon^2)$.

$$\left[\right] = \left[\frac{\sqrt{z^2 + R^2} - z}{R^2 \sqrt{z^2 + R^2}} \right] \approx \left[\frac{z(1 + \frac{R^2}{2z^2}) - z}{R^2 \sqrt{z^2 + R^2}} \right]$$

$$= \left[\frac{1}{2z \sqrt{z^2 + R^2}} \right] \rightarrow \frac{1}{2z^2}$$

$$\therefore \vec{E} \rightarrow \frac{q \hat{z}}{4\pi \epsilon_0 z^2} \text{sgn}(z)$$

WE RECOVER COULOMB'S LAW FOR A POINT CHARGE.

ANOTHER INTERESTING LIMIT: HOLD σ FIXED, $R \rightarrow \infty$ OR $z \rightarrow 0$:

$$\vec{E} = \frac{\sigma \hat{z}}{2\epsilon_0} \text{sgn}(z)$$

INFINITE SHEET OF CHARGE (NO DEPENDENCE ON z !)

WHAT IF $z < 0$? \rightarrow When we remove the constraint $z > 0$, we find a dependence on z through the signum function.