

$$\nabla \times \left(\frac{\vec{A}}{M^2} \right) = 0$$

$$\Rightarrow \boxed{\nabla \times \vec{E} = 0},$$

FOR ALL STATIC FIELDS.

WE WILL LATER LEARN^(NEXT SEMESTER) THAT
TIME-VARYING \vec{B} GIVES RISE
TO $\nabla \times \vec{E} \neq 0$.

CONSEQUENCES: (PER § 1.6)

$$\oint \vec{E} \cdot d\vec{\ell} = 0$$

$\int_a^b \vec{E} \cdot d\vec{\ell}$ IS PATH INDEPENDENT

$$\boxed{\vec{E} = -\nabla V}$$

WE CAN CALL V THE ELECTRIC POTENTIAL.

THE ABOVE DEFINITION ~~LEAVES~~ LEAVES UNDETERMINED THE INTEGRATION CONSTANT, WHICH IS SET BY DECIDING WHERE $V=0$. LET'S FOLLOW GRIFATHS AND CALL THAT SPECIAL LOCATION \vec{Q} :

$$V \equiv - \int_{\vec{Q}}^{\vec{r}} \vec{E} \cdot d\vec{\ell}$$

UNITS: V : VOLTS, $V = J/\text{COUL.}$
 E : $V/m = N/\text{COUL.}$

Q: WHAT ARE THE UNITS OF V ? AND WHY THE MINUS SIGN?

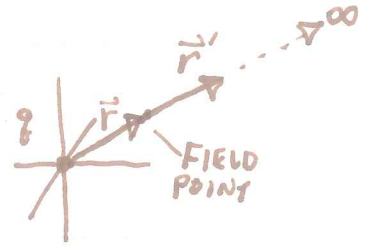
A: COMPARE $U \equiv - \int \vec{F} \cdot d\vec{\ell}$ (POTENTIAL ENERGY)

- UNITS OF V ARE $J/\text{COUL.}$
- MINUS SIGN IDENTIFIES V AS POTENTIAL ENERGY PER CHARGE (RATHER THAN WORK).

EXAMPLE WHAT IS THE POTENTIAL $V(\vec{r})$ FOR A POINT CHARGE q , AT THE ORIGIN?

THE ELECTRIC FIELD IS:

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{\hat{r}}{r^2}$$



$$V(\vec{r}) = -\int_{\vec{0}}^{\vec{r}} \vec{E}(\vec{r}') \cdot d\vec{r}' = \int_{\vec{r}}^{\vec{0}} \vec{E}(\vec{r}') \cdot d\vec{r}'$$

LET $\vec{0}$ BE AT $r \rightarrow \infty$. THIS IS WHERE WE

WILL SET THE POTENTIAL TO ZERO! $d\vec{r}' = \hat{r} dr'$

$$V(\vec{r}) = \int_r^\infty \frac{q}{4\pi\epsilon_0} \frac{dr'}{r'^2} = \frac{-q}{4\pi\epsilon_0} \frac{1}{r'} \Big|_r^\infty = \boxed{\frac{q}{4\pi\epsilon_0 r}}$$

CHECK: $\vec{E} = -\nabla V = -\hat{r} \frac{\partial}{\partial r} \frac{q}{4\pi\epsilon_0 r} = \frac{q\hat{r}}{4\pi\epsilon_0 r^2} \checkmark$

MORE GENERALLY, IF CHARGES q_i ARE LOCATED AT \vec{r}'_i , $V(\vec{r})$ CAN BE EXPRESSED BY SUPERPOSITION:

$$V(\vec{r}) = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i}, \quad r_i \equiv |\vec{r} - \vec{r}'_i| \text{ AS USUAL.}$$

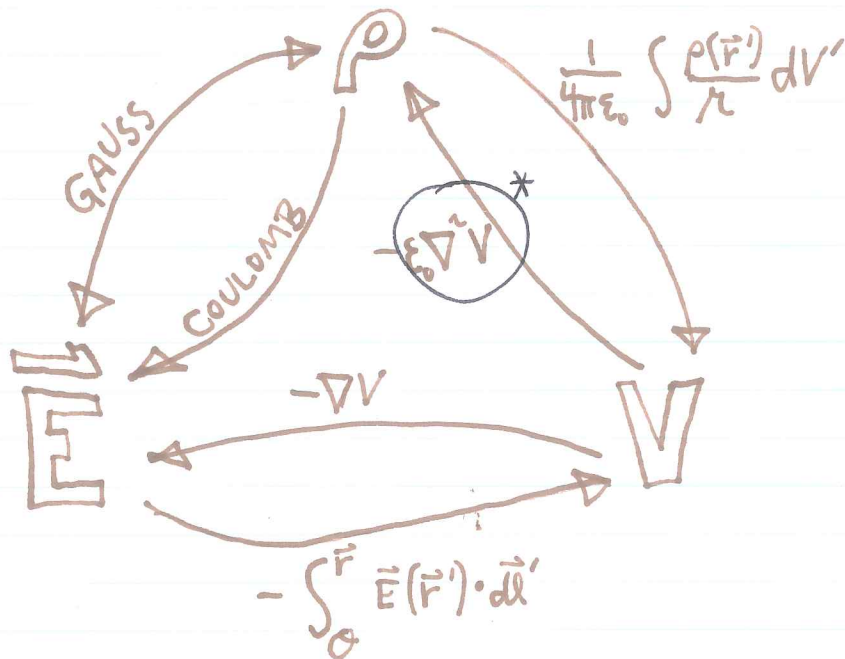
Q: ARE YOU SURE THAT SUPERPOSITION WORKS FOR POTENTIALS, AS WELL AS FOR THE ELECTRIC FIELD?

SIMILARLY, FOR A CHARGE DISTRIBUTION $\rho(\vec{r})$,

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}') dV'}{r}, \quad r = |\vec{r} - \vec{r}'|$$

IT'S LIKE COULOMB'S LAW, BUT EASIER (X3)
SINCE IT IS A SCALAR EXPRESSION.

WHAT WE HAVE SO FAR ARE CALCULATION TECHNIQUES
FOR GOING BETWEEN ρ , \vec{E} , AND V :



$$* \nabla \cdot \vec{E} = \nabla \cdot (-\nabla V) = \rho / \epsilon_0$$

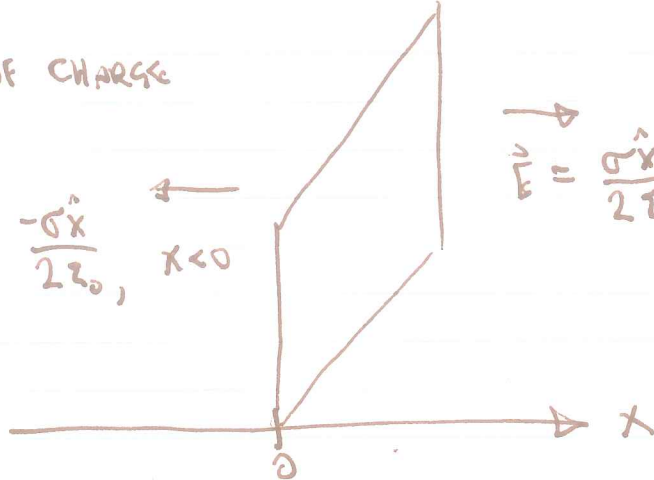
$$\Rightarrow \boxed{\rho = -\epsilon_0 \nabla^2 V}$$

EXAMPLE

∞ PLANE OF CHARGE

$$\vec{E} = -\frac{\sigma x}{2\epsilon_0}, \quad x < 0$$

$$\vec{E} = \frac{\sigma x}{2\epsilon_0}, \quad x > 0$$



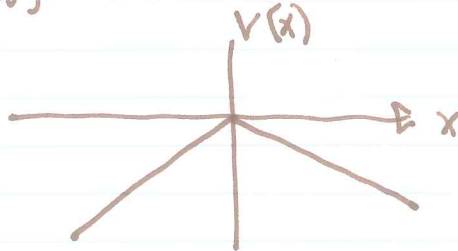
$$x > 0: V = -\int_{\sigma}^x \vec{E} \cdot d\vec{\ell} = -\int_{-\infty}^x E dx' = -\left[\frac{\sigma x'}{2\epsilon_0} - \frac{\sigma \infty}{2\epsilon_0} \right]$$

↑
BAD IDEA!

INSTEAD, LET'S LET $V=0$ @ $x=0$.

$$V = \begin{cases} -\frac{\sigma x}{2\epsilon_0}, & x > 0 \\ \frac{\sigma x}{2\epsilon_0}, & x < 0 \end{cases} = -\frac{\sigma |x|}{2\epsilon_0}$$

DISCUSS



EXERCISE : ∞ LINE CHARGE

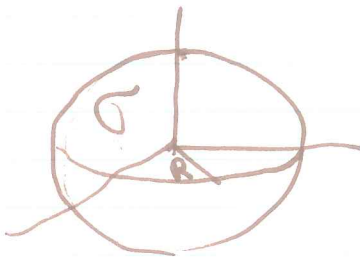
$$V = \frac{-\lambda}{2\pi\epsilon_0} \ln s + \text{CONST.}$$

WHERE DO WE PUT $V=0$??

DISCUSS.

EXAMPLE - SPHERICAL SHELL, CHARGE DENSITY σ
FIND V , \vec{E} , EVERYWHERE.

(5)



Q: WHICH WOULD YOU START WITH, V OR \vec{E} ?

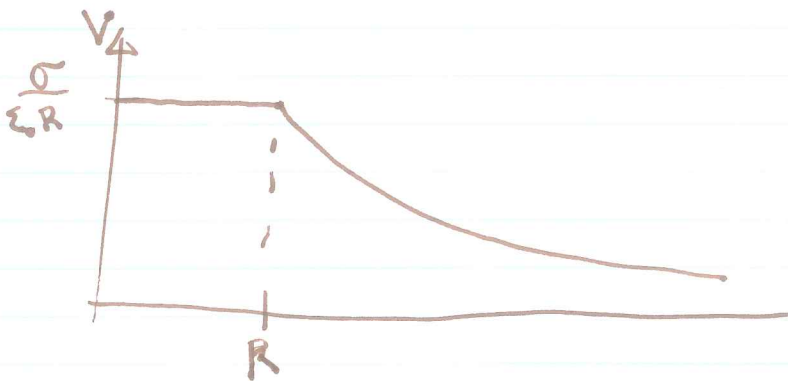
A: GRIFITHS DOES V DIRECTLY, BUT THIS IS A BIT HEROIC.

GAUSS'S LAW GIVES US $\vec{E} = \begin{cases} 0, & r < R \\ \frac{\sigma}{\epsilon_0} \hat{r}, & r > R \end{cases}$

$$\text{THEN } V = - \int_{\infty}^r E(-dr) = \frac{\sigma}{\epsilon_0} \int_r^{\infty} \frac{dr}{r^2} = \frac{\sigma}{\epsilon_0 r}, \quad r > R$$

$$V = \begin{cases} \frac{\sigma}{\epsilon_0 R}, & r < R \\ \frac{\sigma}{\epsilon_0 r}, & r > R \end{cases}$$

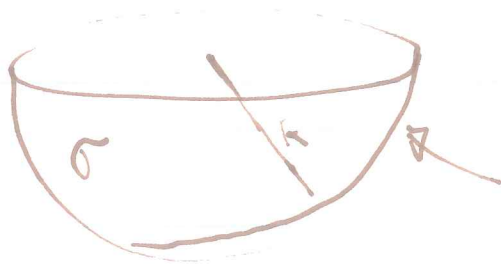
DISCUSS



EXERCISE DO THE SAME FOR A CYLINDRICAL SHELL.

Bowl of Charge

6



HEMISPHERICAL SHELL,
CONSTANT CHARGE DENSITY

Q: HOW MUCH ENERGY DOES IT TAKE TO BRING CHARGE Q FROM ∞ TO THE CENTER OF THE BOWL?

A: QV , WHERE

$$V = \int \frac{dq}{4\pi\epsilon_0 r} = \frac{\sigma \cdot \text{AREA}}{4\pi\epsilon_0 R} = \frac{2\pi R^2 \sigma}{4\pi\epsilon_0 R} = \frac{\sigma R}{2\epsilon_0}$$

DISCUSS

DID THIS APPROACH
SAVE US SOME "WORK"?

Q: WHAT IF WE "FILL" THE BOWL WITH CHARGE DENSITY ρ ?