

EXAMPLE: LINE OF CHARGE, LENGTH a

①

$\vec{r} = s\hat{s} + z\hat{z}$
 $\vec{r}' = s\hat{s} + (z - z')\hat{z}$
 $V(\vec{r}) = \int_{-a/2}^{a/2} \frac{\lambda dz'}{4\pi\epsilon_0 \sqrt{s^2 + z^2 - 2zz' + z'^2}}$

GRC# 237:

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}} \ln(2\sqrt{ac+bcx+c^2x^2} + 2cx+b), \quad c > 0$$

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln(2\sqrt{r^2 - 2zz' + z'^2} + 2(z' - z)) \Big|_{z' = -a/2}^{a/2}$$

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{\sqrt{r^2 - az + a^2/4} + \frac{a}{2} - z}{\sqrt{r^2 + az + a^2/4} - \frac{a}{2} - z} \right]$$

Q: HOW CAN WE CHECK THIS RESULT?

A: LIMITS ~

LARGE r , $V \rightarrow \frac{\lambda a}{4\pi\epsilon_0 r}$ (POINT CHARGE)

SMALL r , $V \rightarrow \frac{-\lambda}{2\pi\epsilon_0} \ln s$ (LINE CHARGE)

(2)

LARGE r LIMIT: (a small)

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{r\sqrt{1 - \frac{az}{r^2} + \frac{a^2}{4r^2}} + \frac{a}{2} - z}{r\sqrt{1 + \frac{az}{r^2} + \frac{a^2}{4r^2}} - \frac{a}{2} - z} \right]$$

$$= \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{r\left(1 - \frac{az}{2r^2}\right) + \frac{a}{2} - z}{r\left(1 + \frac{az}{2r^2}\right) - \frac{a}{2} - z} \right]$$

$$= \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{r - \frac{az}{2r} + \frac{a}{2} - z}{r + \frac{az}{2r} - \frac{a}{2} - z} \right]$$

RECALL
 $\ln(1 + \epsilon) \approx \epsilon$
 $+ \mathcal{O}(\epsilon^2)$

$$= \frac{\lambda}{4\pi\epsilon_0} \ln \left[1 + \frac{-\frac{az}{r} + a}{r + \frac{az}{2r} - \frac{a}{2} - z} \right]$$

$$\approx \frac{\lambda}{4\pi\epsilon_0} \ln \left[1 + \frac{a}{r} \right] \approx \frac{\lambda a}{4\pi\epsilon_0 r} \quad \checkmark$$

THIS LIMIT IS TRICKIER IF $r \rightarrow \infty$ ALONG z -AXIS,
 BUT IT DOES COME OUT THE SAME IN THAT CASE.

(3)

SMALL r LIMIT (a LARGE):

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{\frac{a}{2} \sqrt{1 + \frac{4r^2}{a^2} - \frac{4z}{a}} + \frac{a}{2} - z}{\frac{a}{2} \sqrt{1 + \frac{4r^2}{a^2} + \frac{4z}{a}} - \frac{a}{2} - z} \right]$$

$$= \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{\frac{a}{2} \left(1 - \frac{2z}{a}\right) + \frac{a}{2} - z}{\frac{a}{2} \left(1 + \frac{2z}{a}\right) - \frac{a}{2} - z} \right]$$

$$= \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{a - 2z}{0?} \right]$$

oops! NEED THE r^2 TERM IN THE DENOMINATOR...

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{a - 2z}{\frac{a}{2} \left(1 + \frac{2z}{a} + \frac{2r^2}{a^2}\right) - \frac{a}{2} - z} \right]$$

$$= \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{a - 2z}{r^2/a} \right]$$

IF $z=0$,
 $r=s$

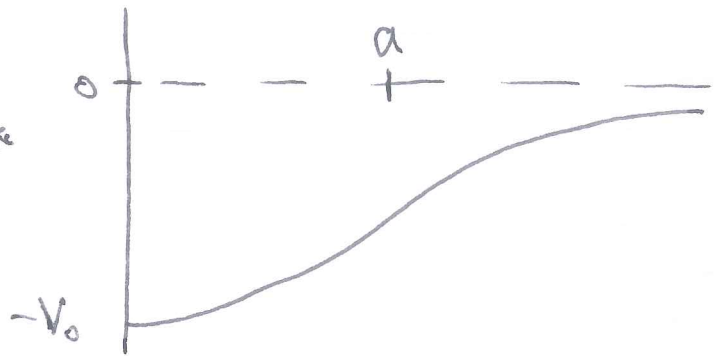
$$= \frac{\lambda}{4\pi\epsilon_0} \left[\cancel{a} - 2\ln s \right]$$

$$= \frac{-\lambda}{2\pi\epsilon_0} \ln s \quad \checkmark$$

SUPPOSE YOU WANTED TO CREATE A POTENTIAL

$$V = -V_0 e^{-r^2/a^2}$$

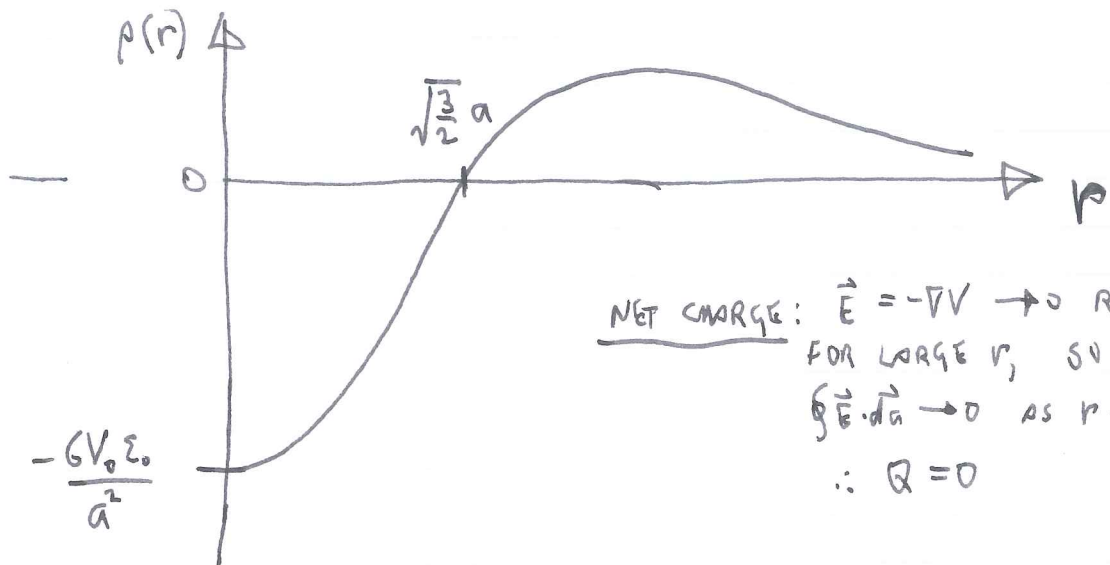
- HOW WOULD YOU ARRANGE THE CHARGE $\rho(\vec{r})$?
- SKETCH $\rho(r)$
- WHAT IS THE NET CHARGE, $\int \rho dV$ OVER ALL SPACE?



$$\nabla^2 V = -\rho/\epsilon_0, \text{ so } \rho = -\epsilon_0 \nabla^2 V$$

$$\begin{aligned} \nabla^2 V &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{2V_0 r}{a^2} e^{-r^2/a^2} \right) \\ &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(\frac{2V_0 r^3}{a^2} e^{-r^2/a^2} \right) = \frac{1}{r^2} \left(\frac{6V_0 r^2}{a^2} - \frac{4V_0 r^4}{a^4} \right) e^{-r^2/a^2} \\ &= \left(\frac{6V_0}{a^2} - \frac{4V_0 r^2}{a^4} \right) e^{-r^2/a^2} \end{aligned}$$

$$\therefore \rho = \frac{2V_0 \epsilon_0}{a^2} \left(\frac{2r^2}{a^2} - 3 \right) e^{-r^2/a^2}$$



NET CHARGE: $\vec{E} = -\nabla V \rightarrow 0$ RAPIDLY
FOR LARGE r , SO
 $\oint \vec{E} \cdot d\vec{a} \rightarrow 0$ AS $r \rightarrow \infty$
 $\therefore Q = 0$