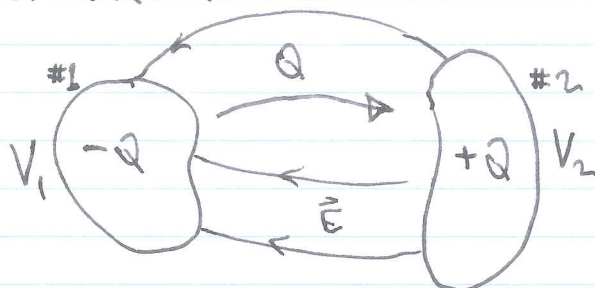


GRIFFITHS §2.5.4 CAPACITORS

CONSIDER 2 CONDUCTORS, INITIALLY UNCHARGED; MOVE CHARGE Q FROM ONE CONDUCTOR TO THE OTHER:



NOW THE CONDUCTORS HAVE EQUAL AND OPPOSITE CHARGES. IF WE CALCULATE

$$V = - \int_{\#1}^{\#2} \vec{E} \cdot d\vec{l} = V_2 - V_1 \quad (> 0 \text{ IF } Q > 0)$$

THEN THE POTENTIAL DIFFERENCE V IS LINEAR IN Q :

$$V \propto Q, \text{ SINCE } \vec{E} \propto Q.$$

WE THEREFORE DEFINE THE CAPACITANCE:

$$\boxed{C = Q/V} \quad (\text{COUL./VOLT} = \text{FARAD})$$

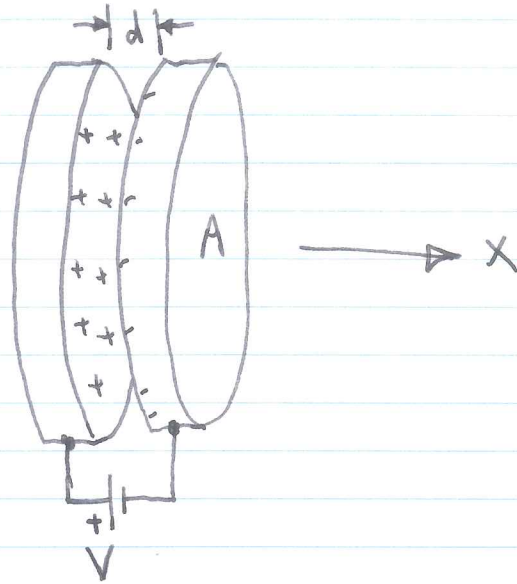
EXAMPLEPARALLEL PLATES OF AREA A:

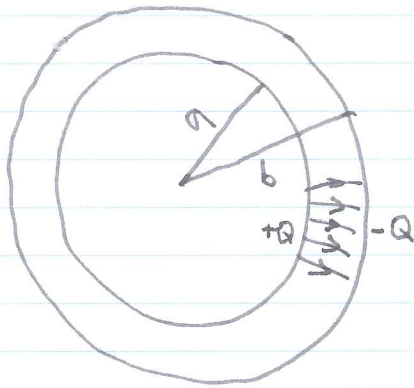
AS LONG AS $A \gg d^2$,
 WE CAN TREAT THESE AS
 INFINITE PLANES: FIELD
 $\vec{E} = E\hat{x}$, CONSTANT AND
 UNIFORM BETWEEN THE
 PLATES.

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$$

$$V = Ed, \text{ SO}$$

$$\frac{V}{d} = \frac{Q}{A\epsilon_0} \Rightarrow \frac{Q}{V} = \boxed{C = \frac{A\epsilon_0}{d}}$$



EXAMPLESPHERICAL SHELLS (GRIFFITHS EX. 2.12)

$$\vec{E} = \frac{Q\hat{r}}{4\pi\epsilon_0 r^2}, \quad V(r) = \frac{Q}{4\pi\epsilon_0 r} + \text{CONST.}$$

↑
DISCUSS

VOLTAGE ACROSS CAPACITOR:

$$V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{Q}{4\pi\epsilon_0} \left(\frac{b-a}{ab} \right)$$

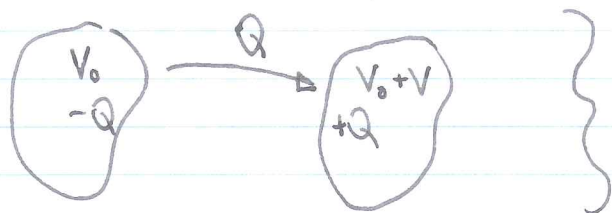
$$\therefore C = \frac{Q}{V} = 4\pi\epsilon_0 \frac{ab}{b-a}$$

COMPARE TO PARALLEL PLATES:

$$\text{LET } d = b - a \ll a$$

$$4\pi ab \approx A$$

$$C = \frac{\epsilon_0 A}{d} \checkmark$$

ENERGY STORED IN A CAPACITOR

ARBITRARY OFFSET,
 V_0
 DEPENDS ON WHERE
 $V=0$.

$$W = \frac{1}{2}(-QV_0 + Q(V_0 + V)) = \frac{1}{2}QV$$

$$Q = CV, \text{ so}$$

$$W = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

(INDEPENDENT OF V_0)

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CHECK: ENERGY OF THE PARALLEL PLATE CAPACITOR

$$W = \frac{1}{2} \epsilon_0 \int E^2 dV$$

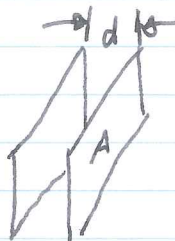
$$= \frac{1}{2} \epsilon_0 E^2 A d$$

$$\text{RECALL } E = \frac{V}{d}$$

$$\therefore W = \frac{1}{2} \epsilon_0 A V^2 / d$$

$$\text{BUT } C = \epsilon_0 A / d, \text{ so}$$

$$W = \frac{1}{2} C V^2 \checkmark$$



EXAMPLE

A PARALLEL PLATE CAPACITOR HAS A 1mm AIR GAP. WHAT VOLTAGE IS REQUIRED FOR THE FORCE PER AREA BETWEEN THE PLATES TO EQUAL 1 ATMOSPHERE ( $10^5 \text{ Pa}$ )?

$$\frac{1}{2} \epsilon_0 E^2 = 10^5 \text{ Pa}$$

$$\begin{aligned} \epsilon_0 &= 8.85 \times 10^{-12} \text{ F/m} \\ &= 8.85 \times 10^{-12} \text{ C/Vm} \end{aligned}$$

$$E = V/d = V/10^{-3} \text{ m}$$

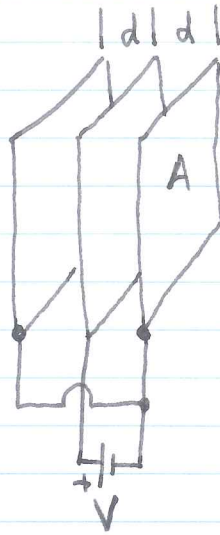
$$V^2 \approx \frac{2 \cdot 10^5 \text{ Pa}}{10^{-11} \text{ C/Vm}} \cdot 10^{-6} \text{ m}^2 \approx 10^{10} \text{ V}^2$$

$$V \approx 10^5 \text{ V} = 100 \text{ kV}$$

FOR COMPARISON, THE DIELECTRIC BREAKDOWN STRENGTH OF AIR IS ABOUT  $3 \text{ MV/m} = 3 \text{ kV/mm}$ .

(6)

EXAMPLE - FIND THE CAPACITANCE.



$$E = V/d = \frac{\sigma}{\epsilon_0}$$

NOTE  $\sigma$  ON BOTH SIDES OF CENTER PLATE.

$$Q = 2\sigma A = \frac{2\epsilon_0 A}{d} V$$

$$\therefore C = \frac{Q}{V} = \frac{2\epsilon_0 A}{d}$$

WHAT'S THE CAPACITANCE IF  $d = 1\text{mm}$  AND  $A = 1\text{m}^2$ ?

$$\frac{2 (10^{-11} \text{ F/m}) 1\text{m}^2}{10^{-3} \text{ m}} = 2 \times 10^{-8} \text{ F} = \textcircled{20 \text{ nF}}$$