

## Review of Ch 2, Electrostatics

FIND  $V(z)$  ALONG THE  $z$ -AXIS

$r = \sqrt{x^2 + z^2}$

$dV = \frac{dq}{4\pi\epsilon_0 r} = \frac{\lambda(x)dx}{4\pi\epsilon_0 \sqrt{x^2 + z^2}}$

$dq = \lambda(x)dx$

SUPPOSE  $\lambda(x) = \begin{cases} \lambda_0 \frac{|x|}{a}, & |x| \leq a \\ 0, & |x| > a \end{cases}$

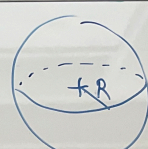
$V(z) = \int dV = \frac{\lambda_0}{4\pi\epsilon_0 a} \int_{-a}^a \frac{|x| dx}{\sqrt{x^2 + z^2}} = \frac{\lambda_0}{2\pi\epsilon_0 a} \int_0^a \frac{x dx}{\sqrt{x^2 + z^2}}$

$= \frac{\lambda_0}{2\pi\epsilon_0 a} \left[ \sqrt{x^2 + z^2} \right]_0^a = \frac{\lambda_0}{2\pi\epsilon_0 a} \left[ \sqrt{a^2 + z^2} - z \right]$

CHECK UNITS!

TRACE INSTRUMENT MECHANICAL LAYOUT

TRA-00101

GIVEN  $\rho = \begin{cases} \frac{A}{r^2}, & r \leq b \\ 0, & r > b \end{cases}$  

FIND:  $\vec{E}(\vec{r}), V(\vec{r})$

SPHERICAL SYMMETRY — USE GAUSS'S LAW:

$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$ , SPHERE OF RADIUS  $R$ ;  $d\tau = 4\pi r^2 dr$

$r \leq b$ :  
 $Q_{\text{enc}} = \int_0^R 4\pi r^2 \left(\frac{A}{r^2}\right) dr = 4\pi A \int_0^R dr = 4\pi AR$

$\oint \vec{E} \cdot d\vec{a} = 4\pi R^2 E = \frac{4\pi AR}{\epsilon_0}$

$\vec{E} = \frac{A\hat{r}}{\epsilon_0 r}$  ← SMALL  $r$

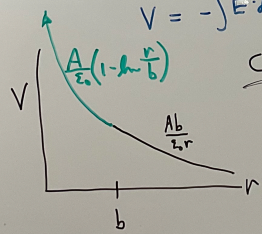
$r > b$ :  $Q_{\text{enc}} = 4\pi Ab$   
 $\vec{E} = \frac{Q_{\text{enc}}\hat{r}}{4\pi \epsilon_0 r^2} = \frac{Ab\hat{r}}{\epsilon_0 r^2}$   
 $V = \frac{Ab}{\epsilon_0 r}$

$V = -\int \vec{E} \cdot d\vec{a} = -\int E dr = -\frac{A}{\epsilon_0} \ln r + C$

$C = ?$  Use  $V = \frac{Ab}{\epsilon_0 r}$  @  $r=b$

$-\frac{A}{\epsilon_0} \ln b + C = \frac{Ab}{\epsilon_0 b} \Rightarrow C = \frac{A}{\epsilon_0} (1 + \ln b)$

$\therefore V = \begin{cases} \frac{A}{\epsilon_0} (1 - \ln \frac{r}{b}), & r \leq b \\ \frac{Ab}{\epsilon_0 r}, & r > b \end{cases}$



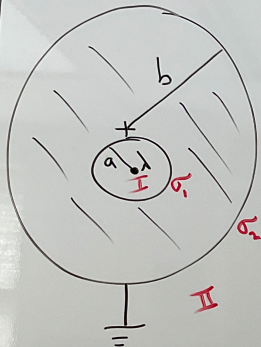
How would you find the energy in the above problem?

$$W = \frac{1}{2} \epsilon_0 \int E^2 d\tau \quad \text{OR} \quad W = \frac{1}{2} \int V dq = \frac{1}{2} \int \rho V d\tau$$

Which is easier?

Only need the inside of the sphere for this one! (for  $\rho \rightarrow 0$  outside)

INFINITE SOLID CONDUCTING CYLINDER  
CONTAINING A CYLINDRICAL HOLE PARALLEL  
TO THE AXIS, AND A LINE CHARGE  $\lambda$ .



FIND  $\sigma$ ,  $\vec{E}$ ,  $V$

$$\sigma_1 = \frac{\lambda}{2\pi a}$$

$$\sigma_2 = 0, \vec{E}_2 = 0$$

GAUSS'S LAW:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$2\pi s \lambda E = \frac{\lambda l}{\epsilon_0}$$

$$\vec{E} = \frac{\lambda s}{2\pi \epsilon_0 s}$$

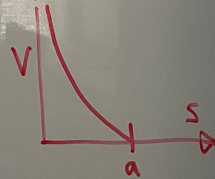
COORDINATES:  
 $\lambda @ s=0$

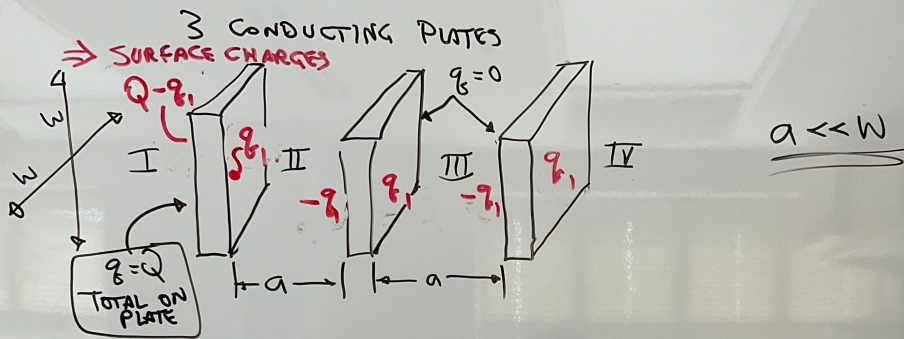


$$V = -\int \vec{E} \cdot d\vec{a} = -\int \frac{\lambda ds}{2\pi \epsilon_0 s} = -\frac{\lambda}{2\pi \epsilon_0} \ln s + C$$

$$V=0 @ s=a: C = \frac{\lambda}{2\pi \epsilon_0} \ln a$$

$$\therefore V = \frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{a}{s}\right)$$





FIND  $\sigma$  ON EACH SURFACE; FIND  $\vec{E}$  IN REGIONS I, II, III, IV.

TO ZERO THE FIELD IN THE LEFT PLATE,

$$Q - q_1 = q_1 \Rightarrow q_1 = \frac{Q}{2}$$

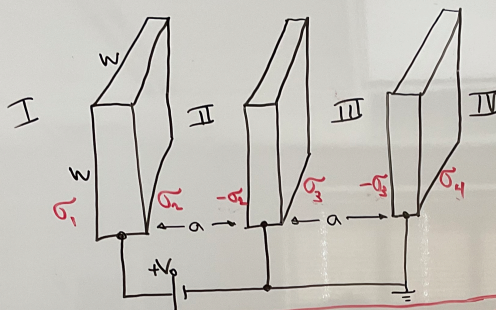
THE SURFACE CHARGES ARE THEREFORE:

$$\frac{Q}{2W^2}, \frac{Q}{2W^2}; \quad \frac{-Q}{2W^2}, \frac{Q}{2W^2}; \quad \frac{-Q}{2W^2}, \frac{Q}{2W^2}.$$

FIELDS:

$$\vec{E}_I = -\frac{Q \hat{x}}{2\epsilon_0 W^2} \quad \vec{E}_{II} = \vec{E}_{III} = \vec{E}_{IV} = +\frac{Q \hat{x}}{2\epsilon_0 W^2}$$

SAME 3 PLATES, BUT CONNECTED TO A BATTERY:



FIND

- SURFACE CHARGE DENSITIES (6)
- FIELDS (4)
- CAPACITANCE

$$\vec{E} = -\nabla V \Rightarrow \boxed{\vec{E}_{II} = \frac{V_0}{a} \hat{x}; \vec{E}_{III} = 0}$$

WE ALSO KNOW (GAUSS'S LAW)  $\vec{E}_{II} = \frac{\sigma_2}{\epsilon_0} \hat{x} \Rightarrow \boxed{\sigma_2 = \frac{V_0 \epsilon_0}{a}}$

$$\vec{E}_{III} = \frac{\sigma_3}{\epsilon_0} \hat{x} \Rightarrow \boxed{\sigma_3 = 0}$$

TO KEEP THE FIELDS IN THE CONDUCTORS ZERO, REQUIRE  $\sigma_1 = \sigma_4$ .

THE ONLY LOGICAL CHOICE IS  $\sigma_1 = \sigma_4 = 0$ . (MINIMIZES ENERGY)

THERE IS A SLIGHT (IGNORABLE) CONTRADICTION HERE. **DISCUSS.**

$$C \equiv \frac{Q}{V} = \frac{\sigma_2 w^2}{V_0} = \frac{V_0 \epsilon_0 w^2}{a V_0} = \boxed{\frac{\epsilon_0 w^2}{a}}$$

How about energy? You could integrate the field, but it's easier to use  $W = CV^2/2$ .