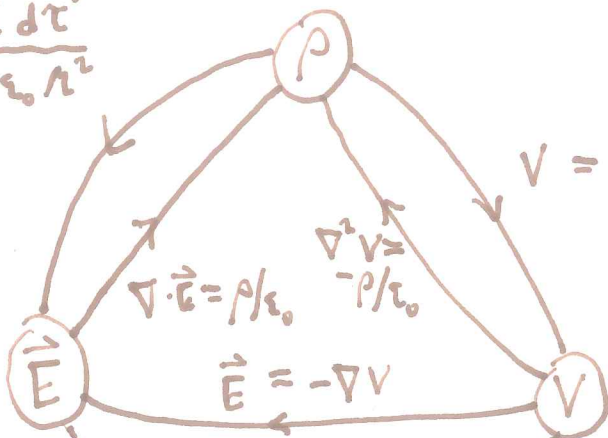


INTRO TO GRIFFITHS CH. 3

WE'VE BEEN DOING ELECTROSTATICS —

$$\vec{E} = \int \frac{\rho \hat{r} d\tau'}{4\pi\epsilon_0 R^2}$$



$$V = \int \frac{\rho d\tau'}{4\pi\epsilon_0 R}$$

$$V(\vec{b}) - V(\vec{a}) = - \int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$$

GAUSS: $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$ / $\nabla \cdot \vec{E} = \rho/\epsilon_0$

FORCES: $\vec{F} = q\vec{E}$, $\vec{F}_{cond.} = \epsilon_0 E^2 = \frac{\sigma^2}{\epsilon_0}$ CONDUCTORS

CAPACITORS: $C \equiv Q/V$

$$W = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

ENERGY: $W = \frac{1}{2} \int \rho V d\tau = \frac{1}{2} \epsilon_0 \int E^2 d\tau$

NOW WE WILL LAYER SOME MATH ON TOP...

IMPORTANCE OF POTENTIAL (VS. \vec{E})

- EASIER TO GET FROM ρ
- CAN EASILY GET $\vec{E} = -\nabla V$
- DIRECTLY RELATED TO ENERGY
- MORE NATURAL IN THE PRESENCE OF CONDUCTORS, WHERE SURFACE POTENTIAL IS CONSTANT ... AS OPPOSED TO σ , WHICH MIGHT HAVE A COMPLICATED DISTRIBUTION.

THIS CHAPTER FOCUSES ON CALCULATING V BY VARIOUS METHODS.

TO BEGIN, CONSIDER:

$$\nabla^2 V = -\rho / \epsilon_0 \quad (\text{POISSON'S EQ.})$$

$$\boxed{\nabla^2 V = 0} \quad \text{IN VACUUM (LAPLACE'S EQ., } \rho = 0)$$

THIS IS ~~AND~~ A PDE, BUT AS WE WILL SEE, WE CAN SOLVE IT IN MANY SITUATIONS AND THE CHARACTER OF THE SOLUTIONS IS FAIRLY INTUITIVE.

LAPLACE EQN. IN 1D

I.E. $\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0!$

CARTESIAN: $\frac{\partial^2 V}{\partial x^2} = 0 \Rightarrow \frac{\partial V}{\partial x} = \text{const.}$

$\Rightarrow V = Cx + D$

$\vec{E} = -C\hat{x}$, UNIFORM FIELD

THIS IS THE SOLUTION WE'VE BEEN USING IN SLAB GEOMETRIES, E.G. PARALLEL PLATE CAPACITOR.

IN ANY CURVILINEAR COORDINATE SYSTEM, WE CAN PICK JUST 1 OF THE 3 COORDINATES TO MAKE A 1-D PROBLEM.

CYLINDRICAL:

THIS ONE IS NO DIFFERENT FROM CARTESIAN.

$0 = \nabla^2 V = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

THIS ONE REPRESENTS THE CYLINDRICAL SYMMETRY WE HAVE BEEN EXPLOITING WITH GAUSS'S LAW:

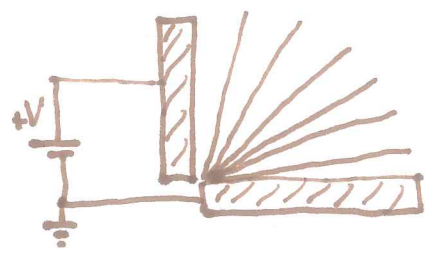
$\frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V}{\partial s} \right) = 0 \Rightarrow s \frac{dV}{ds} = \text{const.}$

$\Rightarrow V = V_1 \ln s + V_0$
(LOOK FAMILIAR?)

THE ϕ -TERM IS FUNNY:

$\frac{1}{s^2} \frac{\partial^2 V}{\partial \phi^2} = 0 \Rightarrow \frac{\partial V}{\partial \phi} = \text{const.} \Rightarrow V = V_1 \phi + V_0$

$\Rightarrow \vec{E} \propto \frac{\hat{\phi}}{r}$



SPHERICAL

$$0 = \nabla^2 V = \underbrace{\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right)}_{\text{WE HAVE SOLVED THE } V(r) \text{ PROBLEM WITH GAUSS'S LAW.}} + \underbrace{\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right)}_{\text{INTERESTING!}} + \underbrace{\frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}}_{\text{NO DIFFERENT FROM CYLINDRICAL CASE (DISCUSS!)}}$$

WE HAVE SOLVED
THE $V(r)$ PROBLEM
WITH GAUSS'S LAW.

INTERESTING!

NO DIFFERENT FROM
CYLINDRICAL CASE
(DISCUSS!)

EXERCISE: WORK OUT A SOLUTION TO $\nabla^2 V = 0$ FOR THE CASE WHERE V DEPENDS ONLY ON θ .