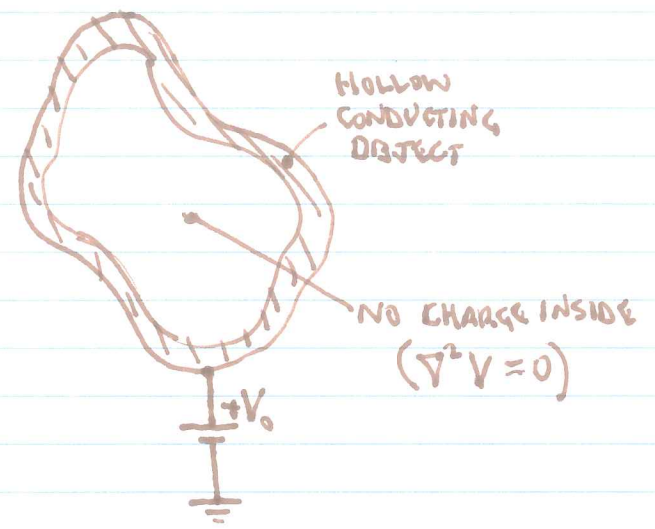


GRIFFITH § 3.1.5

BOUNDARY CONDITIONS AND UNIQUENESS THEOREMS

EXERCISE

FIND V
INSIDE.



SOLUTION: $V = V_0$ ON BOUNDARY.

SUPPOSE $V > V_0$ SOMEWHERE INSIDE. THAT IMPLIES THERE IS A LOCAL MAXIMUM, BUT WE HAVE PROVEN THAT'S IMPOSSIBLE!

SIMILAR ARGUMENT FOR $V < V_0$.

$\therefore V = V_0$ EVERYWHERE INSIDE.

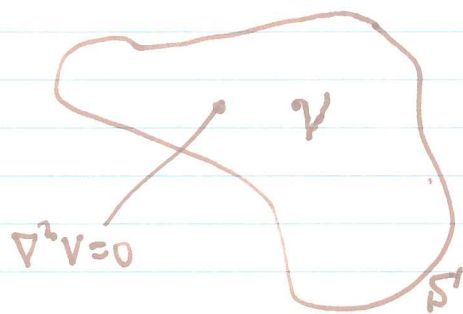


ASSERTION:

"THE SOLUTION TO LAPLACE'S EQUATION ON VOLUME \mathcal{V} IS UNIQUELY DETERMINED BY THE VALUE OF THE POTENTIAL ON THE BOUNDING SURFACE S ."

(FIRST UNIQUENESS THEOREM)

- DISCUSS 2D ANALOGY
- DISCUSS THE "POINT RELAXATION" DEMOS.

PROOF

SUPPOSE WE KNOW $V(\vec{r})$
FOR $\vec{r} \in S$.

NOW SUPPOSE $V_1(\vec{r})$ AND $V_2(\vec{r})$ ARE TWO
INTERIOR SOLUTIONS THAT SATISFY LAPLACE'S EQN.
AND THE BOUNDARY CONDITIONS.

NOW CONSIDER $V_3 \equiv V_1 - V_2$.

$$\nabla^2 V_3 = \nabla^2 V_1 - \nabla^2 V_2 = 0 - 0 = 0$$

$$\text{SINCE } V_1(\vec{r})|_S = V_2(\vec{r})|_S, \quad V_3(\vec{r})|_S = 0.$$

THIS IMPLIES $V_3(\vec{r}) = 0!$

$$\therefore V_1 = V_2.$$

QED

LICENSE TO GUESS & CHECK:

IF YOU CAN GUESS A SOLUTION TO $\nabla^2 V = 0$
AND IT FITS THE BOUNDARY CONDITIONS, THEN
LOOK NO FARTHER!

THIS IS VERY HELPFUL IN CONCEPTUALLY CHALLENGING
SITUATIONS, WHERE YOU HAVE A REASONABLE/EDUCATED
GUESS.

SUPPOSE $\rho \neq 0$ IN THE VOLUME.

$$\nabla^2 V_1 = \nabla^2 V_2 = -\rho/\epsilon_0$$

$$\nabla^2 V_3 = \nabla^2 V_1 - \nabla^2 V_2 = 0; \text{ AGAIN } V_3|_S = 0.$$

$$\therefore V_3 = 0$$

$$\therefore V_1 = V_2$$

COROLLARY : THE POTENTIAL IN \mathcal{V} IS UNIQUELY DETERMINED IF THE POTENTIAL IS KNOWN ON THE BOUNDARY AND THE CHARGE DENSITY INSIDE \mathcal{V} IS ALSO KNOWN.