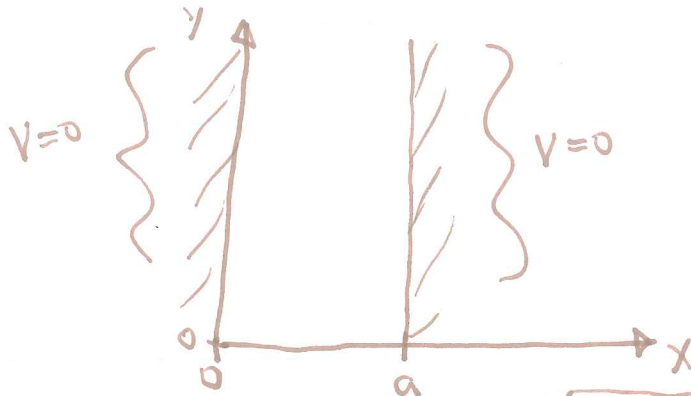


LAST TIME, WE PERFORMED SEPARATION OF VARIABLES TO SOLVE LAPLACE'S EQUATION IN A SEMI-INFINITE NOTCH:



STARTED BY LOOKING AT SOLUTIONS TO $\nabla^2 V = 0$ OF THE FORM:
 $V = X(x)Y(y)$.

$$V = \sum_{n=1}^{\infty} A_n \underbrace{\sin(\pi n x/a)}_{X_n(x)} \overbrace{e^{-\pi n y/a}}^{Y_n(y)}$$

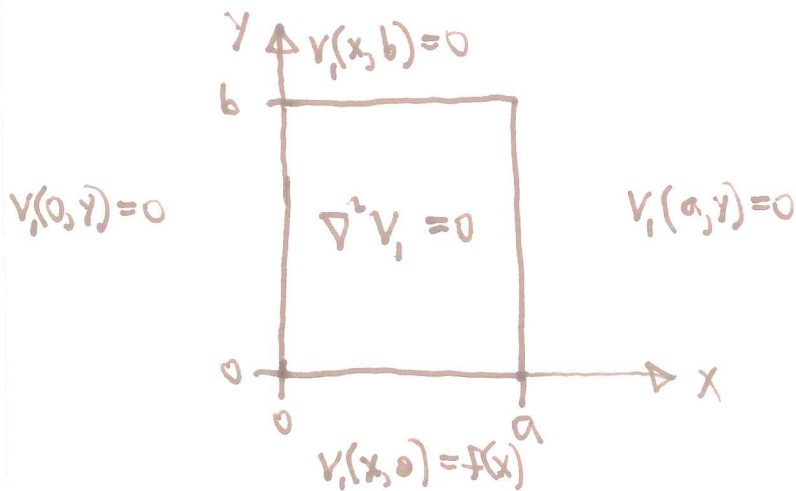
$$A_n = \frac{2}{a} \int_0^a f(x') \sin(\pi n x'/a) dx'$$

CHOOSING PRIMED NOTATION PREVENTS MISUNDERSTANDING IF WE PLUG A_n INTO V .

TODAY I WANT TO DO 3 THINGS:

- I. CHANGE DOMAIN TO $0 < y < b$ (FINITE 2D BOX)
- II. ADD INHOMOGENEOUS BOUNDARY CONDITION TO THE OPPOSITE SIDE OF THE BOX.
- III. DO A 3D PROBLEM.

THE HOMEWORK EXERCISES ALL OF THE TECHNIQUES WE WILL USE TODAY!



I. CLOSED BOX

THE ONLY CHANGE IS THE HOMOGENEOUS DIRICHLET B.C. ON $y=b$.

Q: HOW CAN WE MODIFY OUR SOLUTION?

A: PREVIOUSLY, WE FOUND $Y = A e^{-\pi n y/a} + B e^{+\pi n y/a}$.

WE FOUND $B=0$ FIT OUR $V \rightarrow 0$ AS $y \rightarrow \infty$ REQUIREMENT. BUT NOW, WE WOULD LIKE $V=0$ @ $y=b$:

$$Y(b) = A e^{-\pi n b/a} + B e^{+\pi n b/a} = 0$$

$$B = -A e^{-2\pi n b/a} \quad *$$

Q: WHAT DO FOR $Y_1(0)$?

A: $Y_1(0) = 1$ WAS CONVENIENT LAST TIME (DISCUSS). LET'S STICK WITH THAT!

$$Y_1(0) = A + B = 1 \quad **$$

COMBINE * WITH **:

$$A = 1 / (1 - e^{-2\pi n b/a})$$

$$B = -1 / (e^{2\pi n b/a} - 1)$$

$$Y_1 = \frac{e^{-\pi n (y-b)/a} - e^{+\pi n (y-b)/a}}{e^{\pi n b/a} - e^{-\pi n b/a}}$$

QUICKLY CHECK THAT $Y_1(0) = 1$ AND $Y_1(b) = 0$.

cont. →

THE MAGIC WE DID FOR SATISFYING

$$V(x,0) = \sum_{n=1}^{\infty} A_n \delta_n(x) = f(x)$$

STILL WORKS! SO NOW WE HAVE:

$$V = \sum_{n=1}^{\infty} A_n \sin(\pi n x/a) \left[\begin{array}{cc} e^{-\pi n(y-b)/a} & e^{+\pi n(y-b)/a} \\ e^{\pi n b/a} & -e^{-\pi n b/a} \end{array} \right]$$

WITH A_n AS BEFORE.

II. INHOMOGENEOUS BOUNDARIES ON 2 SIDES

4

Q: WHAT WOULD WE DO IF THE TOP & BOTTOM BOUNDARIES WERE SWITCHED, E.G.

$$\nabla^2 V_2 = 0 \quad \begin{cases} V_2(x, b) = g(x) & V_2(0, y) = 0 \\ V_2(x, 0) = 0 & V_2(a, y) = 0 \end{cases}$$

A: ① IN $\nabla(y)$, LET $y \rightarrow \cancel{y} b - y$

$$\nabla_2 = \frac{e^{\pi ny/a} - e^{-\pi ny/a}}{e^{\pi nb/a} - e^{-\pi nb/a}} \quad \begin{cases} \nabla(0) = 0 \checkmark \\ \nabla(b) = 1 \checkmark \end{cases}$$

$$\textcircled{2} \quad B_n = \frac{2}{a} \int_0^a g(x') \sin(n\pi x'/a) dx'$$

$$\textcircled{3} \quad V_2 = \sum_{n=1}^{\infty} B_n \sin(n\pi x/a) \left[\frac{e^{\pi ny/a} - e^{-\pi ny/a}}{e^{\pi nb/a} - e^{-\pi nb/a}} \right]$$

Q: WHAT ARE THE BOUNDARY VALUES FOR $V_{\text{TOT}} = V_1 + V_2$?

$$A: V_{\text{TOT}}(0, y) = V_{\text{TOT}}(a, y) = 0$$

$$V_{\text{TOT}}(x, 0) = f(x)$$

$$V_{\text{TOT}}(x, b) = g(x)$$

DISCUSS!

LESSON: APPLY ONE INHOMOGENEOUS B.C. AT A TIME, LEAVING THE OTHER BOUNDARIES HOMOGENEOUS. THEN JUST ADD THE SOLUTIONS!

3D EXAMPLE: GRIFFITHS EX. 3.5