

## GRIFFITHS CH. 4

### ELECTRIC FIELDS IN MATTER

MATTER IS MADE OF "CHARGES ON A LEASH"...

IN HOMEWORK, YOU SHOWED THAT A CONDUCTING SPHERE IS POLARIZABLE, AND THAT ITS DIPOLE MOMENT IS PROPORTIONAL TO THE APPLIED FIELD:

$$\vec{p} = \alpha \vec{E}, \quad \alpha \in \mathbb{R}.$$

SOME OBJECTS — LIKE MOLECULES AND ODDLY SHAPED CONDUCTORS — HAVE A MATRIX POLARIZABILITY:

$$\vec{p} = \underline{\underline{M}} \vec{E},$$

AND THAT IS THE MOST GENERAL FORM FOR LINEAR POLARIZABILITY. WE WILL CONSIDER JUST THE SIMPLE CASE OF  $\underline{\underline{M}} = I \alpha$ , A DIAGONAL POLARIZABILITY IN WHICH ALL THE ELEMENTS HAVE A SINGLE VALUE. THIS IS FINE FOR ATOMS AND ANYTHING WITH ENOUGH SYMMETRY.

REMEMBER THAT THE DIPOLE POTENTIAL IS:

$$V = \frac{1}{4\pi\epsilon_0} \frac{\hat{r} \cdot \vec{p}}{r^2}$$

- DIPOLE AT  $\vec{r}'$
- $\vec{r} = \vec{r} - \vec{r}'$  AS USUAL.

## FORCE, TORQUE, AND ENERGY

CONSIDER A CHARGE DISTRIBUTION (CONTINUOUS OR POINT CHARGES) AT THE ORIGIN. THE DIPOLE MOMENT IS

$$\vec{p} = \int \rho(\vec{r}') \vec{r}' d\tau' = \sum_i q_i \vec{r}_i.$$

APPLY AN ELECTRIC FIELD  $\vec{E}$ . THE FORCE ON  $q_i$  IS  $q_i \vec{E}$ , SO THE TORQUE ON THE WHOLE DISTRIBUTION IS

$$\vec{N} = \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times (q_i \vec{E}) = \left( \sum_i \vec{r}_i q_i \right) \times \vec{E}$$

$$\therefore \boxed{\vec{N} = \vec{p} \times \vec{E}}$$

THE TOTAL FORCE IS:  $\vec{F} = \sum_i q_i \vec{E} = q \vec{E}$

BUT THAT'S THE MONOPOLE TERM. IF THE LEADING TERM IS DIPOLE,  $\vec{F} = 0$ . UNLESS WE CONSIDER SPATIAL VARIATION IN  $\vec{E}$ :

$$\text{LET } \vec{E}(\vec{r}) = \vec{E}(0) + (\vec{r} \cdot \nabla) \vec{E}$$

$$\begin{aligned} \text{NOW } \vec{F} &= \sum_i q_i (\vec{r}_i \cdot \nabla) \vec{E} \\ &= \left[ \left( \sum_i q_i \vec{r}_i \right) \cdot \nabla \right] \vec{E} \end{aligned}$$

$$\boxed{\vec{F} = (\vec{p} \cdot \nabla) \vec{E}}$$

GOING BACK TO A CONSTANT  $\vec{E}$ , THE POTENTIAL ENERGY IS

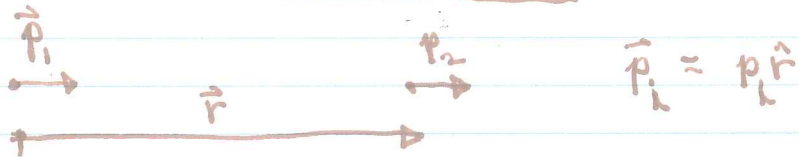
$$U = \sum_i q_i V_i; \quad V(\vec{r}) = -\vec{r} \cdot \vec{E} \quad \text{SO } V_i = V(\vec{r}_i)$$

$$U = \sum_i q_i (-\vec{r}_i \cdot \vec{E}) = \boxed{-\vec{p} \cdot \vec{E}}$$

SO, POLAR MOLECULES (IF FREE TO DO SO) WILL ALIGN THEMSELVES WITH THE FIELD.

EXAMPLE:

FORCE BETWEEN TWO ALIGNED DIPOLES

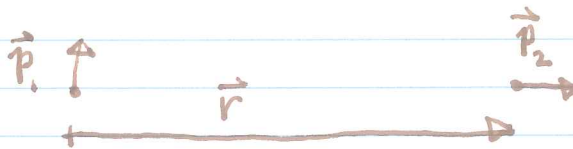


$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0 r^3} [3(\vec{p}_1 \cdot \hat{r})\hat{r} - \vec{p}_1] = \frac{2p_1 \hat{r}}{4\pi\epsilon_0 r^3}$$

$$\vec{F}_2 = (\vec{p}_2 \cdot \nabla) \vec{E}_1 = p_2 \frac{\partial}{\partial r} \left( \frac{\vec{E}_1}{r} \right) = \frac{-3p_1 p_2 \hat{r}}{2\pi\epsilon_0 r^4}$$

EXERCISE

WHAT IS THE FORCE IN THIS CASE?



HOMEWORK INCLUDES PROBLEMS ON POTENTIAL ENERGY AND TORQUE...

GENERALIZATION: FORCE BETWEEN DIPOLES