

GRIFFITHS §4.3

ELECTRIC DISPLACEMENT

IN CH. 3, WE LEARNED ABOUT CONDUCTORS, WHICH ARE CHARACTERIZED BY FREELY MOVABLE CHARGE SUCH THAT $\vec{E} = 0$ INSIDE.

IN THIS CHAPTER, WE HAVE BEEN TALKING ABOUT INSULATORS, WHICH HAVE NO FREE CHARGE. THEY DO, HOWEVER, HAVE ELECTRICAL PROPERTIES BY VIRTUE OF BOUND CHARGE THAT CAN BE POLARIZED. BY VIRTUE OF THIS PROPERTY, INSULATORS ARE ALSO CALLED DIELECTRICS.

IN ANALYZING DIELECTRICS, IT IS CONVENIENT TO DEFINE THE ELECTRIC DISPLACEMENT

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

THE DISPLACEMENT HAS SOME INTERESTING PROPERTIES:

$$\nabla \cdot \vec{D} = \epsilon_0 \nabla \cdot \vec{E} + \nabla \cdot \vec{P} = \rho - \rho_b \equiv \rho_f$$

WE DISTINGUISH "FREE" CHARGE ~~FROM~~ ρ_f AS ALL THE CHARGE THAT ISN'T ~~BOUND~~ PART OF ~~THE~~ MOLECULES IN AN INSULATOR (NOT BOUND).

$$\nabla \cdot \vec{D} = \rho_f$$

SINCE $\nabla \cdot \vec{D}$ INCLUDES ONLY FREE CHARGE, IT IS SOMETIMES EASIER TO CALCULATE THAN \vec{E} .

SAD TO SAY,

$$\vec{D} \neq \frac{1}{4\pi} \int \frac{\rho_f(\vec{r}') \hat{r}}{r^2} d\tau' \quad ! \quad \left(\begin{array}{l} \text{Coulomb's} \\ \text{Law for } \vec{D} \end{array} \right)$$

WHY? NOTE THAT WE USED COULOMB'S LAW TO PROVE $\nabla \times \vec{E} = 0$.

$$\nabla \times \vec{D} = \epsilon_0 \nabla \times \vec{E} + \nabla \times \vec{P} = \nabla \times \vec{P} \neq 0 \quad (\text{IN GENERAL})$$

DISCUSS: $\nabla \times \vec{P}$ FOR A BAR ELECTRO

BOUNDARY CONDITIONS

FROM GAUSS'S LAW, WE KNOW THAT

$$\vec{E}_{\text{OUT}} \cdot \hat{n} = \vec{E}_{\text{IN}} \cdot \hat{n} + \frac{\rho}{\epsilon_0}$$

$$E_{\text{OUT}}^{\perp} - E_{\text{IN}}^{\perp} = \frac{\rho}{\epsilon_0}$$

AND SINCE $\nabla \times \vec{E} = 0$,

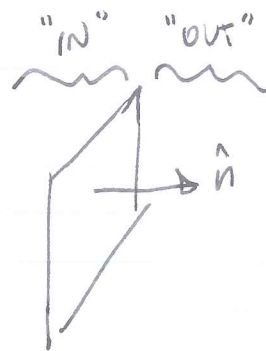
$$\vec{E}_{\text{OUT}}^{\parallel} - \vec{E}_{\text{IN}}^{\parallel} = 0$$

SIMILARLY,

$$P_{\text{OUT}}^{\perp} - P_{\text{IN}}^{\perp} = -\sigma_b$$

$$D_{\text{OUT}}^{\perp} - D_{\text{IN}}^{\perp} = \sigma_f$$

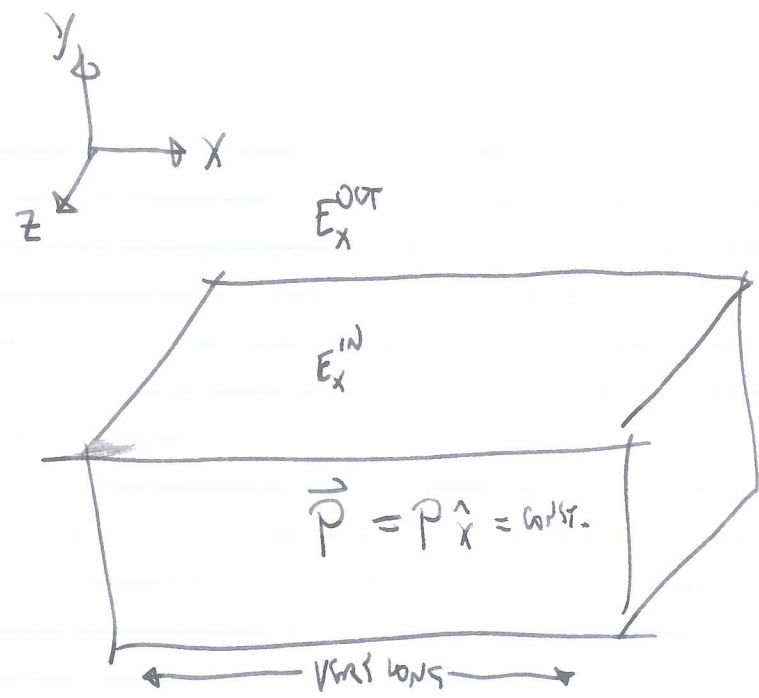
$$\vec{D}_{\text{OUT}}^{\parallel} - \vec{D}_{\text{IN}}^{\parallel} = \vec{P}_{\text{OUT}}^{\parallel} - \vec{P}_{\text{IN}}^{\parallel}$$



ASIDE

IF YOU NEED A FORMAL DEFINITION OF THE PARALLEL FIELDS, TRY THIS OUT:

$$\vec{E}^{\parallel} =$$



$$\vec{D} = ?$$

$$\nabla \times \vec{D} = \nabla \times \vec{P}$$

$$E_x^{OUT} - E_x^{IN} = 0$$

$$D_x^{OUT} - D_x^{IN} = P_x^{OUT} - P_x^{IN} = -P$$

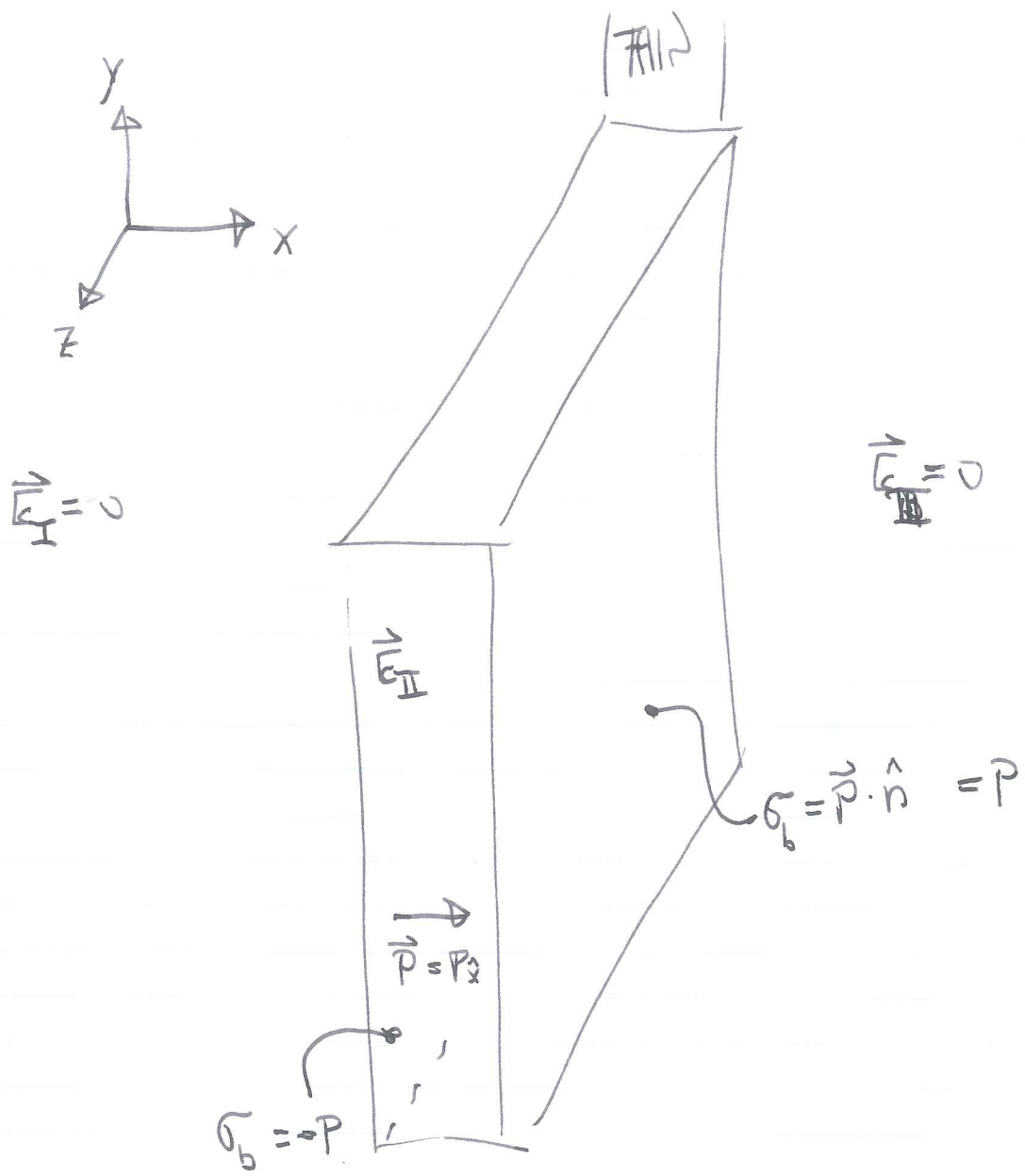
$$D_x^{IN} = P$$

$$D_x^{OUT} = 0$$

$$\epsilon_0 \vec{E} = \vec{D} - \vec{P} = 0 \text{ EVERYWHERE}$$

DISCUSS

Q: WHAT ARE THE FIELDS LIKE NEAR THE ENDS?



$$\vec{E}_I = -\frac{P}{\epsilon_0} \hat{x} \quad (\text{GAUSS})$$

$$\nabla \cdot \vec{D} = \rho_f = 0, \text{ so}$$

$$\vec{D}_I = \vec{D}_{II} = \vec{D}_{III}$$

$\vec{P} = \vec{E} = 0$ OUTSIDE, so $\vec{D} = 0$ OUTSIDE

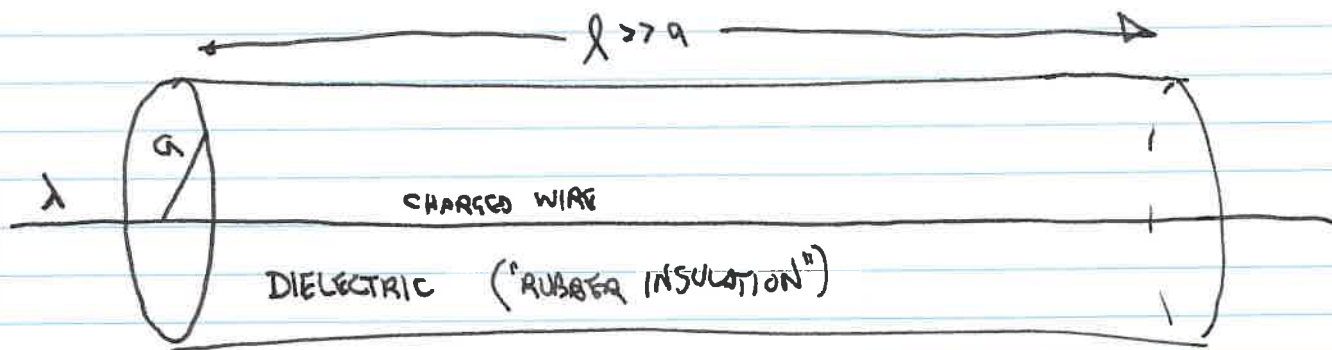
CHECK:

$$\vec{D}_I = \epsilon_0 \vec{E}_I + \vec{P} = 0$$

so $\vec{D} = 0$ EVERYWHERE

Q: WHAT ARE THE FIELDS LIKE NEAR THE EDGES?

GRIFFITHS EXAMPLE 4.4



FIND \vec{D} , IGNORING END EFFECTS.

SOLUTION

GRIFFITHS USES GAUSS'S LAW,

$$\oint \vec{D} \cdot d\vec{a} = Q_{\text{FREE}}$$

SINCE λ IS FREE CHARGE, APPARENTLY

$$\vec{D} = \frac{\lambda \hat{s}}{2\pi s} \quad \text{--- TRUE OUTSIDE ALSO! (DISCUSS)}$$

→ WE CAN ALSO INFER $\vec{E}_{\text{OUT}} = \vec{D}/\epsilon_0$, BUT NOT \vec{E}_{IN} BECAUSE \vec{P} IS UNKNOWN!

NOW CONSIDER THE POSSIBLE EFFECTS OF THE POLARIZATION OF THE DIELECTRIC: 3 CASES OF INTEREST:

(i) $\vec{P} = P(s) \hat{s}$

DOES THIS CHANGE THE ANSWER?

- \vec{D} IS UNCHANGED (GAUSS'S LAW STILL WORKS!)

- $\vec{E} = (\vec{D} - \vec{P})/\epsilon_0$ (EASY!)

(ii) $\vec{P} = P_0 \hat{z}$ (CONSTANT, UNIFORM)

- $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, SO $\vec{D}_{\text{IN}} = \frac{\lambda \hat{s}}{2\pi s} + \vec{P}$.

- $\vec{E} = (\vec{D} - \vec{P})/\epsilon_0 = \frac{\lambda \hat{s}}{2\pi \epsilon_0 s}$ --- BOTH INSIDE & OUT

(iii) $\vec{P} = \frac{s}{a} P_0 \hat{\phi}$?? (SIMILAR TO ii)

GRIFFITHS IGNORES ANY POSSIBILITY OF POLARIZATION ALONG \hat{z} OR $\hat{\phi}$ IN HIS EXAMPLE. WHY DO YOU SUPPOSE HE DOES THAT?