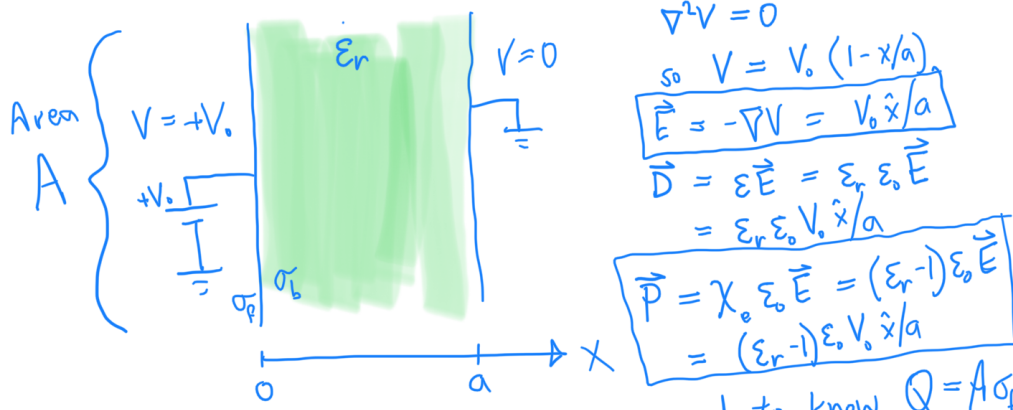


# Energy in Linear Dielectrics

§ 4.4.3

Consider the parallel plate capacitor with a dielectric:



To find the capacitance, we need to know  $Q = A\sigma_p$ :

$$\sigma \equiv \sigma_b + \sigma_p; \quad \vec{E} = \frac{\sigma}{\epsilon_0} \hat{x} \text{ (Gauss's law)}$$

$$\Rightarrow \sigma = \epsilon_0 E = \epsilon_0 V_0/a$$

Now,  $\sigma_b = -(\epsilon_r - 1) \epsilon_0 V_0/a < 0$  (Tries to cancel  $\sigma_p$ )

$$\therefore \sigma_p = \sigma - \sigma_b = \frac{\epsilon_0 V_0}{a} (1 - -(\epsilon_r - 1)) = \frac{\epsilon_r \epsilon_0 V_0}{a}$$

$$\Rightarrow Q = \frac{A}{a} \epsilon_r \epsilon_0 V_0$$

$$\therefore C = \frac{Q}{V_0} = \frac{\epsilon_r \epsilon_0 A}{a}$$

This is how the dielectric affected the capacitance.

Stored energy:

$$W = \int_0^a V dq = \frac{1}{2} V_0 Q = \frac{1}{2} C V_0^2 \text{ (As derived previously)}$$

$$= \frac{\epsilon_r \epsilon_0 A V_0^2}{2a} = \frac{1}{2} \epsilon_r \epsilon_0 E^2 \cdot Aa \text{ (Recall } E = \frac{V_0}{a})}$$

We infer

$$W = \int \frac{1}{2} \epsilon E^2 d\tau = \frac{1}{2} \int \vec{D} \cdot \vec{E} d\tau \text{ (Griffiths eq. 4.58)}$$