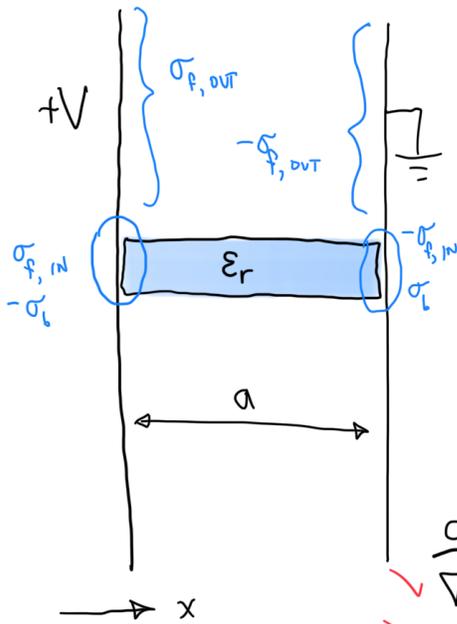


Example

A rectangular block of insulator, dielectric constant ϵ_r , is inserted into a parallel plate capacitor as depicted below. The capacitor is held at constant voltage. Solve for \vec{E} , \vec{P} , and \vec{D} inside and outside the capacitor.



Ansatz: $\vec{E} = \frac{V}{a} \hat{x}$ both inside and outside the insulator.

If my guess above is correct, then

$$\vec{P}_{in} = \epsilon_0 \chi_e \vec{E}_{in} = \epsilon_0 (\epsilon_r - 1) \frac{V}{a} \hat{x}$$

$$\vec{D} = \epsilon \vec{E}, \text{ so}$$

$$\vec{D}_{in} = \epsilon_r \epsilon_0 \vec{E} = \epsilon_r \epsilon_0 \frac{V}{a} \hat{x}$$

$$\vec{D}_{out} = \epsilon_0 \frac{V}{a} \hat{x}$$

check:

$$\checkmark \nabla^2 V_{in} = \nabla^2 V_{out} = 0$$

$$\checkmark \vec{E}_{in}^{\parallel} = \vec{E}_{out}^{\parallel}$$

$$\checkmark \vec{D}_{in}^{\parallel} - \vec{D}_{out}^{\parallel} = \vec{P}_{in}^{\parallel} - \vec{P}_{out}^{\parallel}$$

Surface charges:

$$\sigma_{P,OUT} = \epsilon_0 E = \epsilon_0 \frac{V}{a}$$

$$\sigma_b = \vec{P}_{in} \cdot \hat{x} = \epsilon_0 (\epsilon_r - 1) \frac{V}{a}$$

Since $\vec{E}_{in} = \vec{E}_{out}$,

$$\sigma_{P,IN} - \sigma_b = \sigma_{P,OUT} \Rightarrow \sigma_{P,IN} = \epsilon_0 \epsilon_r \frac{V}{a}$$

If we imagine an infinitesimal gap between the plate and the insulator,

$$\vec{E}_{gap} = \epsilon_r \frac{V}{a} \hat{x}$$

★ Now, are the conditions on \vec{E}^{\perp} and \vec{D}^{\perp} satisfied?

