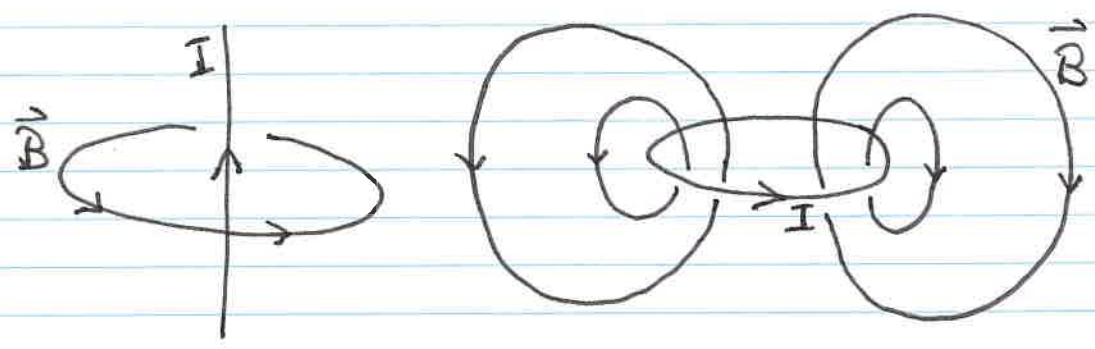


# GRIFFITHS CH. 5 — MAGNETOSTATICS

## §5.1 QUALITATIVE:



- MAGNETIC FIELDS ARE PRODUCED BY MOVING CHARGES (CURRENTS).
  - RIGHT HAND RULE
  - WE WILL DEVELOP THE QUANTITATIVE RELATIONSHIP BETWEEN CURRENT AND FIELD IN §5.2-5.3.
  - BUT FIRST, WE'LL LOOK AT FORCE EXERCISED BY THE MAGNETIC FIELD,  $\vec{B}$ .
- BIOT-SAVART LAW      AMPERE'S LAW  
 ↑                            ↑

LORENTZ FORCE LAW GIVEN A PRE-EXISTING  $\vec{B}$ , ...

$$\vec{F}_M = q \vec{v} \times \vec{B} \quad (\text{MAGNETIC FORCE})$$

$$\vec{F} = \vec{F}_E + \vec{F}_M = q [\vec{E} + \vec{v} \times \vec{B}]$$

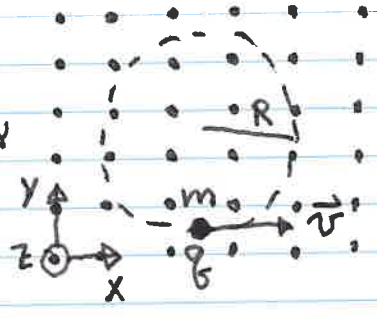
- MOVING CHARGE  $\Rightarrow \vec{B} \Rightarrow$  FORCE ON MOVING CHARGE
- "MAGNETOSTATICS" MEANS NOT THAT CHARGES AREN'T MOVING, BUT RATHER THAT

$$\frac{\partial \vec{B}}{\partial t} = 0 \quad \text{AND} \quad \frac{\partial \rho}{\partial t} = 0.$$

EXAMPLE: CYCLOTRON MOTION

SUPPOSE  $\vec{B} = B \hat{z}$  (UNIFORM FIELD),  $\vec{v} = v \hat{x}$  INITIALLY

THE FORCE  $q \vec{v} \times \vec{B}$  IS ALWAYS PERPENDICULAR TO  $\vec{v}$ , SO IT DOES NO WORK\* AND CONSEQUENTLY THE PARTICLE IS DEFLECTED BUT NEITHER SPEEDS UP NOR SLOWS DOWN. THE RESULT IS UNIFORM CIRCULAR MOTION.



CENTRIPETAL FORCE:  $\frac{mv^2}{R} = qvB$

$\therefore R = \frac{mv}{qB}$

\*  $dW = \vec{F}_m \cdot d\vec{r} = \vec{F}_m \cdot \vec{v} dt$   
 $= q(\vec{v} \times \vec{B}) \cdot \vec{v} dt = 0.$

MAGNETIC FORCES DO NO WORK!

EXAMPLE — A SPECIAL CASE OF " $\vec{E} \times \vec{B}$ " DRIFT,  
WHICH IS IMPORTANT FOR IONS AND  
ELECTRONS IN EARTH'S MAGNETOSPHERE.

CAN THE ELECTRIC AND MAGNETIC FORCES BE BALANCED,  
RESULTING IN NO ACCELERATION ( $\vec{v} = \text{CONSTANT}$ )?

$$q [\vec{E} + \vec{v} \times \vec{B}] = 0 \quad \text{BUT } q \neq 0 \quad (q = 0 \text{ IS NOT INTERESTING!})$$

$$\therefore \vec{E} = -\vec{v} \times \vec{B} \quad (\text{THE VALUE OF } q \text{ IS NOT IMPORTANT!})$$

$$\Rightarrow \vec{E} \cdot \vec{v} = 0, \quad \vec{E} \cdot \vec{B} = 0$$

$$\text{LET US CONSIDER } \vec{B}_\perp \equiv \vec{B} - \frac{(\vec{B} \cdot \vec{v})\vec{v}}{v^2}$$

(COMPONENT OF  $\vec{B}$  PERPENDICULAR TO THE MOTION),

$$\text{BECAUSE } \vec{v} \times \vec{B} = \vec{v} \times \vec{B}_\perp, \quad \cancel{\vec{v} \times \frac{(\vec{B} \cdot \vec{v})\vec{v}}{v^2}}$$

NOW, WE HAVE

$$\vec{E} = -\vec{v} \times \vec{B}_\perp$$

$$\Rightarrow \vec{E} \times \vec{B}_\perp = \vec{B}_\perp \times (\vec{v} \times \vec{B}_\perp) \quad (\text{LHS - BY SWITCHING ORDER})$$

$$= \vec{v}(\vec{B}_\perp \cdot \vec{B}_\perp) - \vec{B}_\perp(\vec{B}_\perp \cdot \vec{v}) = \vec{B}_\perp v$$

$$\therefore \vec{v} = \frac{\vec{E} \times \vec{B}_\perp}{B_\perp^2}$$

$$\text{AND } |\vec{v}| = v = \frac{E}{B_\perp}$$

GRIFFITHS (EX. 5.2) FINDS THE SAME AVERAGE VELOCITY  
FOR A CHARGED PARTICLE EXECUTING CYCLOIDAL  
MOTION IN UNIFORM FIELDS  $\vec{E}, \vec{B}$ , WITH  
 $\vec{E} \perp \vec{B}$  AND BOTH PERPENDICULAR TO  $\langle \vec{v} \rangle_{\text{AVG}}$ .

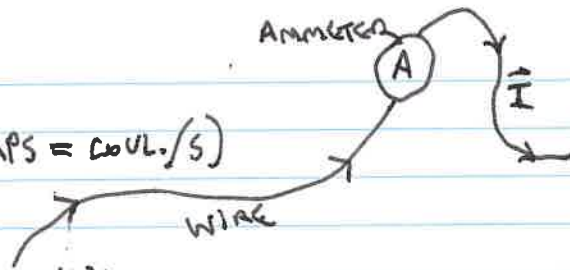
→ WHAT HAPPENS IF  $q$  BECOMES  $-q$ ? SKETCH.

CURRENT

$$I \equiv \frac{dq}{dt} \quad (\text{UNITS: AMPS} = \text{COUL.}/\text{S})$$

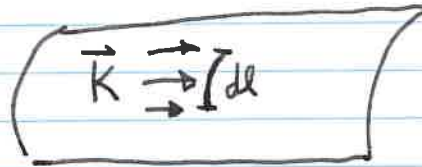
"CHARGE PASSING THE AMMETER," OR  
SOME OTHER POINT ALONG THE WIRE.

WE NORMALLY ASSUME  $|I|$  IS CONSTANT ALONG THE  
WIRE ( $\partial\rho/\partial t = 0$ ). DIRECTION FOLLOWS THE WIRE.

SURFACE CURRENT DENSITY:

$$\vec{K} \equiv \frac{d\vec{I}}{d\ell} \quad \dots \quad d\ell \text{ IS A PATH CUTTING ACROSS } \perp \text{ TO THE CURRENT.}$$

UNITS: A/m



$$\vec{J} \equiv \frac{d\vec{I}}{dA} \quad \dots \quad \text{CHARGE } \text{PASSING THROUGH A SURFACE } \perp \text{ TO THE CURRENT.}$$



BUT OFTEN, CURRENTS ARE IN NEUTRAL CONDUCTORS.

• EQUAL, OPPOSITE CHARGE DENSITIES

$\rho$  ... CHARGE CARRIERS (ELECTRONS)

$-\rho$  ... MATRIX, NOT MOVING (IONS)

EXAMPLE SURFACE CHARGE  $\sigma$  ON SPINNING RECORD (LP).

CONSIDER THE INTERVAL  $dl$ ,  
FROM  $s$  TO  $s+dl$ , RADIALLY.  
THE CHARGE  $\sigma dl$  MOVES AT VELOCITY  
SPEED  $\vec{v} = \omega s \hat{\phi}$ ; SPEED  $v = \omega s$ ,  
PERPENDICULAR TO THE LINE ELEMENT  
 $dl$ . SO CHARGE



$dq = \sigma dl \cdot v dt$   
GOES BY PER UNIT TIME. THIS IS A CURRENT

$$dI = \frac{dq}{dt} = \sigma v dl = \sigma \omega s dl.$$

$$K = \frac{dI}{dl} = \sigma v = \sigma \omega s.$$

$$\vec{K} = \sigma \vec{v} = \sigma \omega s \hat{\phi}.$$

WHAT IS THE TOTAL CURRENT GOING AROUND THE DISK?

$$I = \int_0^R K ds = \sigma \omega \cdot \frac{R^2}{2} = \underline{\underline{\frac{1}{2} \sigma \omega R^2}}.$$

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FORCE ON A CURRENT

WHAT IS THE CURRENT DUE TO A MOVING LINE CHARGE?



$$\vec{I} = \frac{dq}{dt} \hat{x} = \lambda \frac{dx}{dt} = \lambda \vec{v}$$

LIKELIKE,

$$\vec{K} = \sigma \vec{v} \quad (\text{AS SEEN IN THE EXAMPLE!})$$

AND

$$\vec{J} = \rho \vec{v}$$

WHAT ARE:

$$\lambda \vec{v} \times \vec{B} \quad \text{--- FORCE PER LENGTH}$$

$$\sigma \vec{v} \times \vec{B} \quad \text{--- FORCE PER AREA}$$

$$\rho \vec{v} \times \vec{B} \quad \text{--- FORCE PER VOLUME}$$

$$\therefore \vec{F}_m = \int I (d\vec{x} \times \vec{B})$$

$$\vec{F}_m = \int dV (\vec{J} \times \vec{B})$$

$$\vec{F}_m = \int dA (\vec{K} \times \vec{B})$$

Q: ★ WHAT IS THE FORCE PER UNIT LENGTH ON A WIRE CARRYING CURRENT  $I$  IN A  $\perp$  PERPENDICULAR MAGNETIC FIELD?

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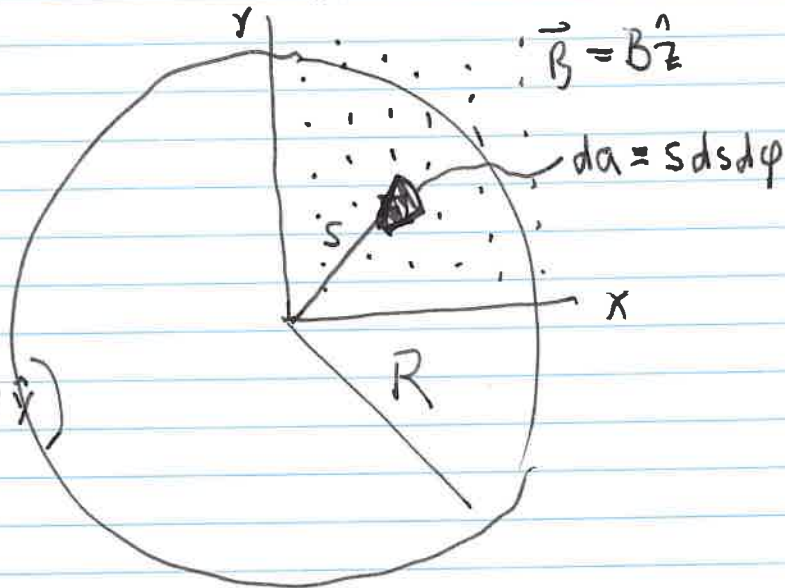
WHAT IF THE UPPER RIGHT QUADRANT ( $x > 0, y > 0$ ) ONLY OF OUR DISK (LP RECORD) WERE IN A MAGNETIC FIELD? WHAT WOULD BE THE FORCE ON THE DISK?

$$d\vec{F}_m = \vec{K} \times \vec{B} da;$$

$$= \sigma \omega s \hat{\phi} \times \hat{z} B$$

$$= B \sigma \omega s \hat{s}$$

$$= B \sigma \omega s (\cos \phi \hat{x} + \sin \phi \hat{y})$$



$$\vec{F}_m = \int d\vec{F}_m$$

$$= \int_{s=0}^R ds \int_{\phi=0}^{\pi/2} d\phi [B \sigma \omega s (\cos \phi \hat{x} + \sin \phi \hat{y})]$$