

PREPARATION FOR THE HOMEWORK (#1):
 EIGENFUNCTIONS OF d^2/dx^2 , B.C.'S PERIODIC IN
 THE LIMIT OF $a \rightarrow \infty$, $-a < x < a$.

- YOU MIGHT NOTICE THAT FOR ~~MANY~~ "MOST" $w(x)$,
 THE SOLUTION TO $y'' = w(x)$ CANNOT SATISFY
 THE BOUNDARY CONDITIONS. TRY NOT TO LET
 THIS WORRY YOU!
- EIGENFUNCTIONS WITH POSITIVE EIGENVALUES,
 E.G. e^{ax} , $a \in \mathbb{R}$, CLEARLY DON'T WORK.
- LOOK AT $y = e^{ikx}$.

FOR FINITE a , THE ALLOWED WAVELENGTHS ARE

$$k_n = \frac{\pi n}{a}$$

THE SPACING BETWEEN ALLOWED k DECREASES
 TO ZERO AS $a \rightarrow \infty$:

$$\Delta k = k_{n+1} - k_n = \frac{\pi}{a}$$

\therefore ALL e^{ikx} SATISFY THE B.C.'S AS $a \rightarrow \infty$.

SO WE ARE JUST USING THE FOURIER BASIS!

$$\langle x | k \rangle = \frac{1}{\sqrt{2\pi}} e^{ikx}$$

*E.G., POLYNOMIALS IN x .

PREPARATION FOR THE HOMEWORK (#2): MAKING A BASIS OUT OF SPHERICAL BESSEL FUNCTIONS n_0 & j_0 :

IT IS FAIRLY OBVIOUS THAT $\langle n_0 | j_0 \rangle = 0$, BUT THE NORMALIZATION INTEGRAL $\langle n_0 | n_0 \rangle$ OR $\langle j_0 | j_0 \rangle$ DIVERGES. THIS IS ANALOGOUS TO WHAT HAPPENED WITH THE FOURIER BASIS (HW 1, PROBLEM 2).

NOTE THAT $n_0(kr) = \frac{\sin kr}{kr}$, $j_0(kr) = \frac{\cos kr}{kr}$.

Actually, $j_0(kr) = \sin(kr)/kr$; $n_0(kr) = -\cos(kr)/kr$. This error is inconsequential though.

~~error~~ $\langle n_{0,k} | n_{0,k'} \rangle = \frac{1}{kk'} \int_0^\infty \sin kr \sin k'r dr$
 $= \lim_{\alpha \rightarrow 0} \frac{1}{kk'} \int_0^\infty e^{-\alpha r} \sin kr \sin k'r dr$

$= \lim_{\alpha \rightarrow 0} \frac{1}{kk'} \int_0^\infty e^{-\alpha r} \left(\frac{-i}{2}\right) (e^{ikr} - e^{-ikr}) \left(\frac{-i}{2}\right) (e^{ik'r} - e^{-ik'r}) dr$

$= \lim_{\alpha \rightarrow 0} \frac{1}{4kk'} \left[\frac{-+1}{i(k+k')-\alpha} + \frac{- -1}{i(k-k')-\alpha} + \frac{- -1}{-i(k-k')-\alpha} + \frac{-+1}{-i(k+k')-\alpha} \right]$

$= \lim_{\alpha \rightarrow 0} \frac{+1}{4kk'} \left[\frac{2\alpha}{(k-k')^2 + \alpha^2} + \frac{-2\alpha}{(k+k')^2 + \alpha^2} \right]$

NOW, NOTE THAT $\int_{-\infty}^\infty \frac{\alpha dx}{\alpha^2 + x^2} = \text{TAN}^{-1} \frac{x}{\alpha} \Big|_{-\infty}^\infty = \pi$

$\Rightarrow \lim_{\alpha \rightarrow 0} \frac{\alpha}{\alpha^2 + x^2} = \pi \delta(x)$

$\therefore \langle n_{0,k} | n_{0,k'} \rangle = \frac{\pi}{2kk'} (\delta(k-k') - \delta(k+k'))$

THE PROPER DOMAIN IS $k > 0$ (AND $k' > 0$), SO THE 2nd TERM IS IGNOREABLE.

PROPERLY NORMALIZED: $k \sqrt{\frac{2}{\pi}} n_0(kr)$ (SIMILARLY FOR $j_0(kr)$)