

1

WE'LL NOW DO A SERIES OF EXAMPLES INVOLVING INTEGRATION OF TIME SERIES DATA. THE SAME CONCEPTS COME INTO PLAY WITH INTEGRATION OF IMAGE DATA. SEE FOR EXAMPLE DEFOREST & KANKELBORG, arXiv:0704.2058 [astro-ph].

- FIRST, LOOK AT THE ELEVATOR ACCELERATION EXAMPLE (accel.m, position.m).
- HOW DOES ERROR ACCUMULATE WHEN WE INTEGRATE ACCELERATION TO GET VELOCITY? (SEE error-trend.m).

REPRODUCING THE TREND:

SUPPOSE WE HAVE N MEASUREMENTS, $a_i \pm \sigma$.
ASSUME FIXED TIMESPAN T AND UNCERTAINTY σ .

INTEGRATE a_i TO GET FINAL v :

$$v_f = \left(\sum_i a_i \right) \Delta t, \quad \text{WHERE } \Delta t = T/N.$$

CUMULATIVE ERROR:

$$\Delta v = \Delta t \sqrt{\sum_i \sigma^2} = \frac{\sigma T}{\sqrt{N}}$$

~~Q~~ Q: HOW WOULD THE ERROR SCALE WITH T IF WE HELD Δt FIXED?

Q: IS THERE ANY ADVANTAGE TO USING A HIGHER ORDER INTEGRATION ROUTINE LIKE SIMPSON'S RULE OR TRES HERMANOS?

IS THERE ANY WAY TO AVOID THIS MOUNTING ERROR?

IT IS INEVITABLE, UNLESS WE HAVE SOME PRIOR KNOWLEDGE....

CONSIDER AN OPTICAL TABLE. IF WE INTEGRATE ACCELEROMETER DATA OVER A LONG TIME AND FIND THAT THE TABLE HAS WANDERED NEXT DOOR, THAT CONTRADICTS WHAT WE ALREADY KNOW ABOUT THE TABLE: IT'S JUST SITTING IN THE LAB!

→ LOOK AT THE INTEGRAL FILTER DERIVATION ON THE NEXT PAGE.

→ THEN LOOK AT THE EXAMPLES:

`ifilter.m`

`ifilter_test.m`

`s2_tailoff.m`

LET $a(t)$ VARY WITH A MEAN OF 0 OVER TIME INTERVALS $\Delta t \gg \tau$.

$$v(t) = \int_{t_0}^t a(t') dt', \quad v(t_0) = 0.$$

BUT IT ISN'T OBVIOUS WHEN t_0 IS, BECAUSE $a(t)$ IS ALL THE INFORMATION WE HAVE ABOUT v .

WHAT IF WE AVERAGE OVER A RANGE OF POSSIBLE t_0 :

$$v(t) = \frac{1}{\tau} \int_{t_0=t-\tau}^t \int_{t'=t_0}^t a(t') dt' dt_0.$$

RE-EXPRESS THE INNER INTEGRAL BY USING A HEAVISIDE FUNCTION TO REMOVE t_0 FROM THE LOWER LIMIT:

$$v(t) = \frac{1}{\tau} \int_{t_0=t-\tau}^t \int_{t'=t-\tau}^t H(t'-t_0) a(t') dt' dt_0.$$

NOW SWITCH THE ORDER OF INTEGRATION:

$$v(t) = \frac{1}{\tau} \int_{t'=t-\tau}^t a(t') \int_{t_0=t-\tau}^t H(t'-t_0) dt_0 dt'$$

↑ THIS CAME OUTSIDE BECAUSE IT CAN.

THE INTEGRAL OVER t_0 IS EASY:

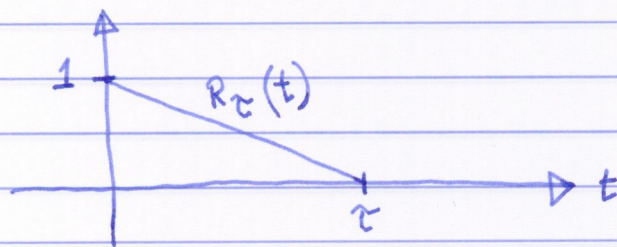
$$\begin{aligned} v(t) &= \frac{1}{\tau} \int_{t'=t-\tau}^t a(t') \left[H(t'-t_0) t_0 \right]_{t_0=t-\tau}^t dt' \\ &= \frac{1}{\tau} \int_{t'=t-\tau}^t a(t') \left[t_0 \right]_{t-\tau}^{t'} dt' \end{aligned}$$

$$\therefore v(t) = \frac{1}{\tau} \int_{t'=t-\tau}^t a(t') [t' - t + \tau] dt'. \quad \star$$



NORMALIZED RAMP FUNCTION OF WIDTH τ :

$$R_\tau(t) = \begin{cases} 1 - t/\tau, & 0 < t < \tau; \\ 0, & \text{ELSE.} \end{cases}$$



Now, $R_\tau(t-t') = \begin{cases} \frac{1}{\tau}(t'-t+\tau), & t-\tau < t' < t; \\ 0, & \text{ELSE.} \end{cases}$

BY COMPARISON WITH \star ,

$$v(t) = \int_{-\infty}^{\infty} R_\tau(t-t') a(t') dt'$$

$$v(t) = R_\tau * a(t) \quad (2)$$

NOTE THE NORMALIZATION OF R_τ :

$$\int_{-\infty}^{\infty} R_\tau(t) dt = \tau/2.$$

WE STARTED WITH

$$v(t) = \int_{t_0}^t a(t') dt', \quad t_0 < t,$$

BUT JUST AS GOOD WOULD BE

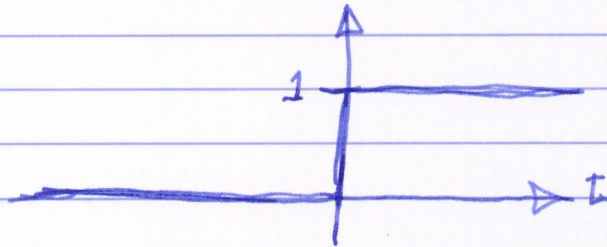
$$v = -\int_t^{t_0} a(t') dt', \quad t_0 > t,$$

WHICH LEADS TO $v = -R_\tau(-t) * a(t)$.

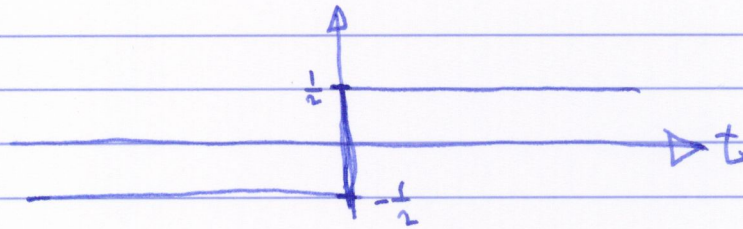
I PROPOSE: $v(t) = \frac{1}{2} [R_\tau(t) - R_\tau(-t)] * a$. (3)

ALTERNATE DERIVATION → FOR THE VISUAL LEARNERS:

INTEGRATION FROM $t' = -\infty$ TO t IS ACCOMPLISHED BY CONVOLUTION WITH HEAVISIDE FUNCTION:

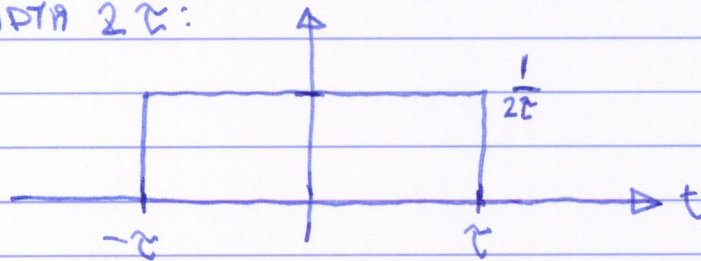


WE CAN ~~NOT~~ SPLIT THE DIFFERENCE $\frac{1}{2} \int_{-\infty}^t + -\frac{1}{2} \int_t^{\infty}$ USING $\frac{1}{2}$ SIGNUM FUNCTION:

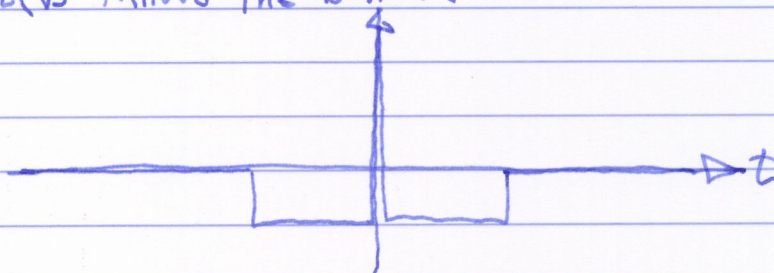


HAVING INTEGRATED, WE WANT TO SUBTRACT A LOCAL MEAN (WHICH WE EXPECT TO BE ZERO).

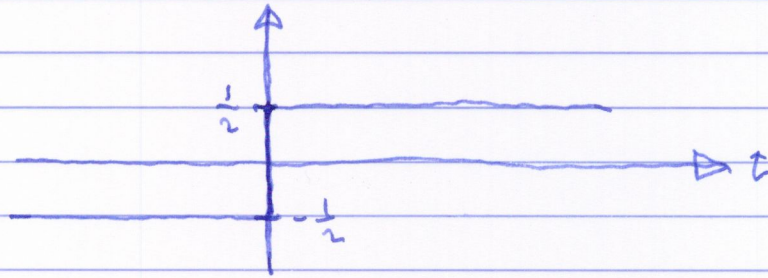
THIS LOCAL MEAN IS REPRESENTED BY A BOXCAR OF WIDTH 2τ :



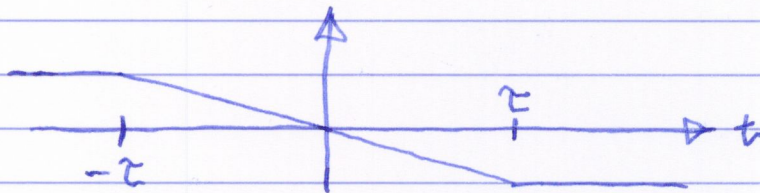
~~WE CAN~~ TO FORM ONE OPERATOR THAT ACCOMPLISHES EVERYTHING, WE CONVOLVE OUR HALF-SIGNUM WITH A $\delta(t)$ MINUS THE BOXCAR:



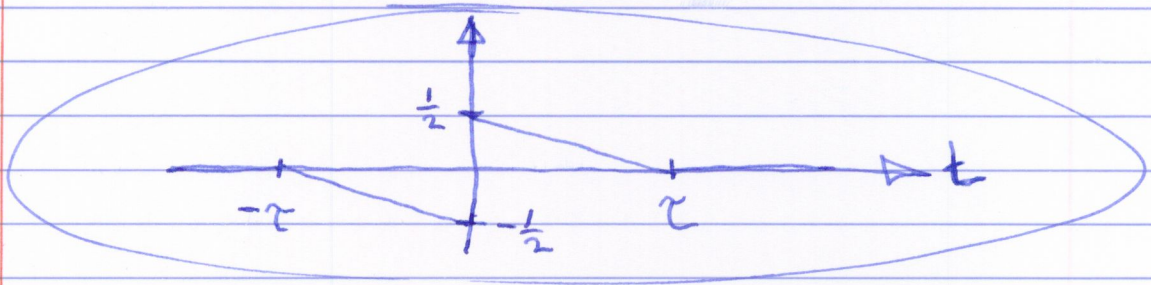
THE RESULT IS THE HALF-SIGNUM PLUS A "DOWNRAMP":



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WHICH IS EXACTLY THE FILTER KERNEL ON
PAGE (4) (EQ. 3).