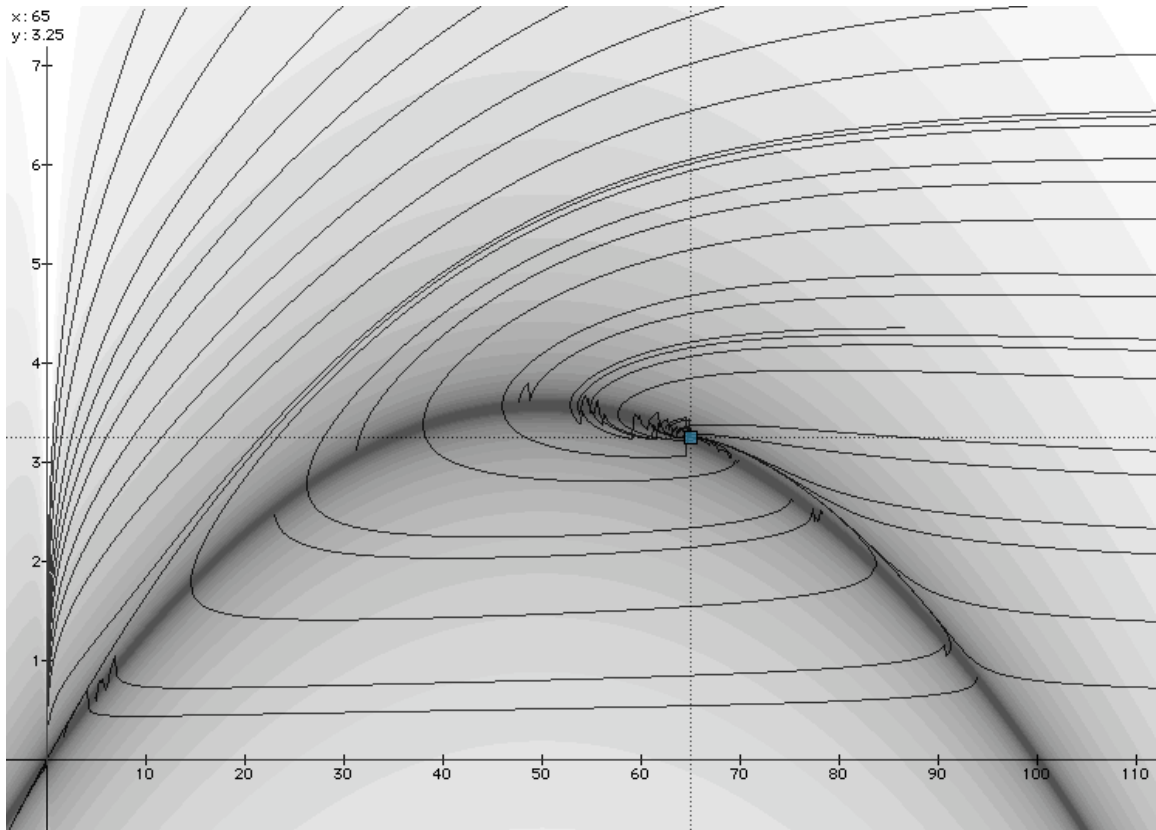


Rabbit hutch ecology

x = rabbit population; y = fox population; t = time in weeks

$$\frac{d}{dt} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x - 0.01x^2 - 7y \\ 0.1y - \frac{2y^2}{x} \end{bmatrix}$$



Assumptions:

- (1) Average one litter of 8 kits per doe per month. Believe it or not, this is in the realm of possibility (barely).
- (2) Rabbit natural attrition rate proportional to x^2 and scaled for an equilibrium population of 100 rabbits in the absence of predation.
- (3) Predation at the rate of one rabbit per fox per day.
- (4) Foxes reproduce at 1/10 the rate of rabbits.
- (5) Fox death rate goes as y^2/x and is scaled to give an equilibrium population of 5 foxes if the rabbit population were held steady at 100.

Speed along trajectory:

Since the phase space plot does not indicate speed of travel, I'll plot that as an image intensity. By way of interpretation, the system tends to "linger" in the region that is darkly shaded.

$$z = \log \left[\begin{array}{l} x - 0.01x^2 - 7y \\ 0.1y - \frac{2y^2}{x} \end{array} \right]$$

Things to notice:

- (1) The jagged phase space trajectories indicate that Graphing Calculator is having a hard time with the integration of the ODEs. That's because the system is stiff.
- (2) There is a stationary ($d/dt = 0$) solution at 65 rabbits, 3.25 foxes. This solution is stable because the phase space trajectories lead inward toward it from all directions.
- (3) There is another stable, stationary solution at (0,0). The paths leading to this solution have the following significance: If the initial fox population is too high, they eat all the rabbits and then die themselves.