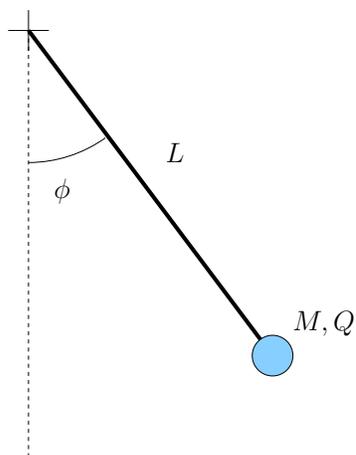


Physics 567
Homework 4
Due Wednesday, March 25, 2026

Chaos in a Driven Pendulum



A massless rod of length L pivots on an imperfect bearing at one end. At the other end is a mass M , with electric charge Q . This pendulum is at rest until time $t = 0$, when a horizontal, uniform, oscillating electric field is turned on:

$$\mathbf{E} = E_0 \hat{x} \cos(\omega t).$$

The equation of motion of the pendulum is

$$\frac{d^2\phi}{dt^2} = -\frac{g}{L} \sin \phi - \mu \frac{d\phi}{dt} + \frac{QE_0}{mL} \cos \phi \cos(\omega t),$$

where μ represents bearing friction.

Depending upon the frequency and amplitude of the electric field, the pendulum will sometimes exhibit chaotic behavior. For example, try the following parameters:

$$\begin{aligned}
M &= 0.0005 \text{ kg} \\
g &= 9.81 \text{ m s}^{-2} \\
\mu &= 0.1 \text{ s}^{-1} \\
L &= 0.1 \text{ m} \\
Q &= 10^{-8} \text{ C} \\
E_0 &= 10^6 \text{ V m}^{-1} \\
\omega &= 11 \text{ rad s}^{-1}
\end{aligned}$$

Plot $\phi(t)$, $0 < t < 30$ s, using the following initial conditions:

1. $\phi = 0, \dot{\phi} = 0$,
2. $\phi = 10^{-4}, \dot{\phi} = 0$.

The two trajectories are initially similar but then diverge, indicating a strong sensitivity to initial conditions.

Your task is now to repeat this experiment, varying E_0 and ω to determine the region in parameter space for which the system shows sensitivity to initial conditions. To perform this task efficiently, you will need to develop a way to quantify sensitivity to initial conditions (it doesn't have to be binary). Your search should cover the range $0 \leq \omega \leq 30$ rad/s and $0 \leq E_0 \leq 1.0$ MV/m, keeping other parameters as listed in the table above.¹ Your result will be a map of the 2D parameter space, indicating the locations where chaotic behavior was detected.

Please turn in the following electronically:

- Your source code, with clear comments.
- A plot of $\phi(t)$, $0 < t < 30$ s, for initial conditions (1) and (2) on the same axes. Use the parameters specified in the table.
- A map delineating sensitivity to initial conditions over the 2D parameter space, searched at whatever resolution you can achieve in a reasonable amount of time.²

In all homework assignments, I encourage you to make use of existing code including my examples and built-in functions in Octave, Matlab, or Python.

¹Electric fields much greater than 1 MV/m may arc in air, bringing an abrupt end to our experiment.

²Even a 10×10 grid is adequate, but the pattern becomes more interesting at higher resolution.