Total solar irradiance correlated with sunspot area, sunspot number, and group sunspot number at different temporal scales using wavelet analysis

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Influence of solar activity cycles on Earth's climate

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http://www.isac-esa.org/
ISAC Database (6 of 28)

- **TSI**: Composite record (Fröhlich and Lean, 1998).
  ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SOLAR_IRRADIANCE/composite_d25_07_0310a.dat

- **SSA**: Sunspot area (Steinegger et al., 1996; Györi, 1998).
  http://science.nasa.gov/ssl/pad/solar/greenwch/daily area.txt

- **Rz**: Solar Influences Data Analysis Centre (SIDC).
  http://sidc.oma.be/DATA/dayssn import.dat

- **Rg**: Group sunspot number (Hoyt and Schatten, 1992, 1998).

- **MWSI**: Mt. Wilson Sunspot Index (Parker et al., 1997; Ballester et al., 2004) (B>100 Gauss).
  ftp://howard.astro.ucla.edu/pub/obs/mpsi data/index.dat

- **MPSI**: Magnetic Plage Strength Index (10<B<100 Gauss).
**Main question**

How is TSI related to solar indices on different temporal scales?

(But also other parameters within ISAC.)

- For high resolution (days, months) the darkening due to sunspots is present in the data and can be modelled by location, area and contrast, and facular brightening using Ca K (Lean et al. 1998).

- For long-term reconstruction (SSA, Rz, Rg only) using temporal averaging, or preferably low-pass filtering, the brightening of TSI may be modelled due to a statistical relation between SSA and facular brightening.

- 2nd order fit of Rz and TSI + slow component (2 month running averages) + fast component (difference) (Solanki and Fligge, 1999).

- TSI reconstruction using different approaches depending on short term or long term (Lockwood, 2005).

- Relation to SSA via finite impulse response filter transformation (Preminger and Walton, 2007).
TSI vs. Rz, daily values

\[ S = 1365.43 + 1.61 \cdot 10^{-2} R_z - 5.5 \cdot 10^{-5} R_z^2 \]

Temporal separation

Slow component (faculae):
\[ \langle \Delta S_{\text{obs}} \rangle \]

two month running means

Fast component (sunspots):
\[ \Delta S_{\text{obs}} - \langle \Delta S_{\text{obs}} \rangle \]

Solanki and Fligge, 1999.
Compensate for autocorrelation when computing confidence limits [Quenouille, 1952; Wilks 1995; von Storch, 1999].
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MODWT

Maximal Overlap Discrete Wavelet Transform

\[ \{x(t)\} \rightarrow \{W_j(t), V(t)\} \quad j = 1, 2, \ldots, J \]

Energy decomposition:  \[ \sum_t x^2(t) = \sum_j \sum_t W_j^2(t) + \sum_t V^2(t) \]

Additive decomposition:  \[ x(t) = \sum_{j=1}^{J} D_j(t) + A(t) \]

Averages:  \[ \langle D_j(t) \rangle = 0 \quad , \quad \langle A(t) \rangle = \langle x(t) \rangle \]

Width of filter:  \[ L_j = (2^j - 1)(L - 1) + 1 \]

[Percival and Walden, 2002]
Wavelet variance

Level \( j \) is associated with periods \( [2^j, 2^{j+1}] \).

Variance at level \( j \):
\[
\nu_j = \frac{1}{N} \sum_t W_j^2(t) \approx 2 \int_{1/2^{j+1}}^{2^j} S(f) df
\]
Scale-based correlation

\[ C_j = \text{CORR}(W_j^x, W_j^y) \]

excluding the

\[ L_j = (2^j - 1)(L - 1) + 1 \]

points affected by the boundary.

95% confidence interval is computed taking into account the autocorrelation in the time series.
Sunspot blocking
Sunspot blocking

Faculae and bright network

TSI and SSA
Sunspot blocking

Faculae and bright network

(Physics)

(Statistics)
Reconstruction on details and approximations separately

\[ TSI_R = a_0 + \sum_{j=J}^{12} a_{j-J+1}D_j + a_{12-J+1}A_{12} \]
Other indices
\[ R_z = k(10g + n) \]

Blocking but weaker than SSA
No blocking
How does this fit with this?

**PSI ∝ SSA [Lockwood, 2005]**

“Darkening by sunspots is quantified by PSI.”
Conclusions

- All parameters correlate around 11 year scales => Makes physical interpretation difficult.

- Among the studied parameters TSI shows the most complex correlation over scales from days to years.

- SSA (MWSI, Rz) and TSI are significantly anti-correlated at scales of 1 to 4 months.

- For reconstruction on short scales (<1.4 years) SSA and MWSI works best, followed by Rz.

- Rg and MPSI are uncorrelated with TSI on scales <1.4 years.

- The statistical confidence is weak on scales above the 11-year variability due to the (limited) extent of TSI.
Extra material
Wavelet variance monthly data
Wavelet power

Power spectrum of monthly Rz

Frequency (year\(^{-1}\))

Power

10^{-2} 10^{-1} 10^{0} 10^{1}

100 1 0.1 10

1/11 y\(^{-1}\) 1/5.3 y\(^{-1}\) 1/2.7 y\(^{-1}\)

9 7 6 5 4 3 2 1
Details at levels 4 and 11
Details at levels 4 and 11

Level 4 [16,32] days

Level 11 [5.6,11.2] years
Details at levels 4 and 11
Details at levels 4 and 11
DETAILS AT LEVELS 4 AND 11

Level 4 [16,32] days

TSI

SSA

Rz

MPSI

WWSI

Jun Jul Aug Sep Oct Nov Dec Jan

1989

Level 11 [5.6,11.2] years

Bolometric brightness

Figure 9 in Lean et al. (1998) shows the ACRIM II brightness with the sunspot darkening removed (grey line) together with the reconstructed brightness from Ca K. Carefully studying the figure shows that the Ca-reconstructed brightness consistently underestimates the ACRIM-SSA brightness. Coupling to SSA-TSI wavelet correlation??

\[ \Delta S_{FAC} = S - S_Q - S_Q \sum_{i=1}^{N} \frac{\mu(3\mu + 2)}{2} \times A_{WDC} [0.2231 + 0.0244 \log_{10}(A_{WDC})] \]
Level 6 coefficients, [64,128] days

\[ C = -0.79 \quad [-0.83, -0.75] \]
Level 11 coefficients, [5.6, 11.2] years

C = 0.98 [0.90, 1.00]
Scaling coefficients, $>22.4$ years

\[ C = 0.93 \ [-0.28, 1.00] \]
\[ x(t) = \sum_{j=1}^{J} D_j(t) + A(t) \]
Coeff. level 11

Details level 11
Scaling coeff.

Approximation
Reconstruction at different levels of approximations

\[ TSI_J = TSI_R + \epsilon \]

\[ TSI_R = a_0 + a_1 A_J \]
Approximation level 3 (>16 days)

$TSI_R = 1365.88 + 1.1 	imes 10^{-4} \times SSA (W/m^2)$

$max(TSI_R) = 1366.69 \text{ in 1947}$

$min(TSI_R) = 1365.88 \text{ in 1876}$
Approximation level 8 (>1.4 years)

\[ TSI_R = 1365.51 + 4.1 \times 10^{-4} \times SSA \ (W/m^2) \]

\[
\begin{align*}
\text{max}(TSI_R) &= 1366.95 \text{ in 1957} \\
\text{min}(TSI_R) &= 1365.51 \text{ in 1913}
\end{align*}
\]
Details 4 to 12 and approximation (>16 days)

max(\(T\!S\!I_R\)) = 1367.52 in 1958

min(\(T\!S\!I_R\)) = 1364.15 in 2003