



Detecting Eccentric White Dwarf Binaries with LISA

Part 1: Investigation of Eccentric System SNRs

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Background

The background of the slide is a 3D visualization of a gravitational well. It features a blue grid that curves downwards to form a deep, circular well. At the center of the well, two bright blue, glowing spheres are positioned close together, representing a binary system. The overall scene is set against a dark blue background with a subtle grid pattern.

- **What is a gravitational wave?**
- **Why do we care about WD binaries?**
- **What role will LISA play?**

Our Motivation

The background of the slide is a 3D visualization of a gravitational well, represented by a blue grid that curves downwards to form a deep well. Two bright blue spheres, representing stars, are positioned within the well, illustrating a binary system. The overall aesthetic is scientific and futuristic.

- **Eccentricity is a natural result due to perturbation**
- **Little literature considering binary WD eccentricity**
- **More physical models can help us better prepare**
- **Help Lane develop the skills necessary to do contemporary astrophysics research**

Objectives

The background of the slide is a 3D visualization of a gravitational well, represented by a blue grid that curves downwards to form a central depression. Two bright blue spheres are positioned within this well, representing a binary system. The overall aesthetic is scientific and futuristic.

- **Part 1 - Investigate the SNR of eccentric WD binaries over a large parameter space to quantify the detectability of such systems (This Talk)**
- **Part 2 - Model the WD confusion noise coming from a galactic population of binary WDs assuming different eccentricity distributions**

Methods



- **All coding done in Python**
- **Libraries include Numpy, Scipy, and Matplotlib**
- **Grounded in previously established literature**
- **Write some codes and do a lot of math**

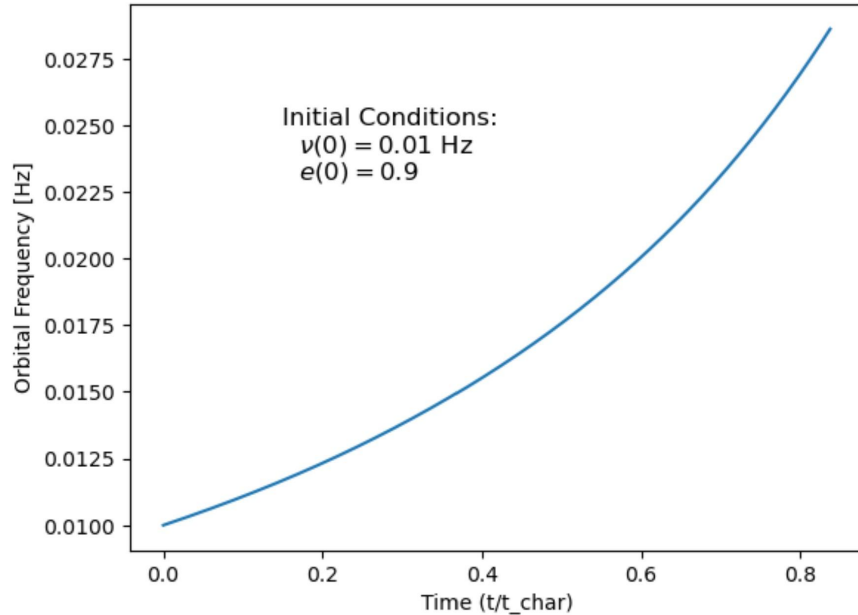
Road-map



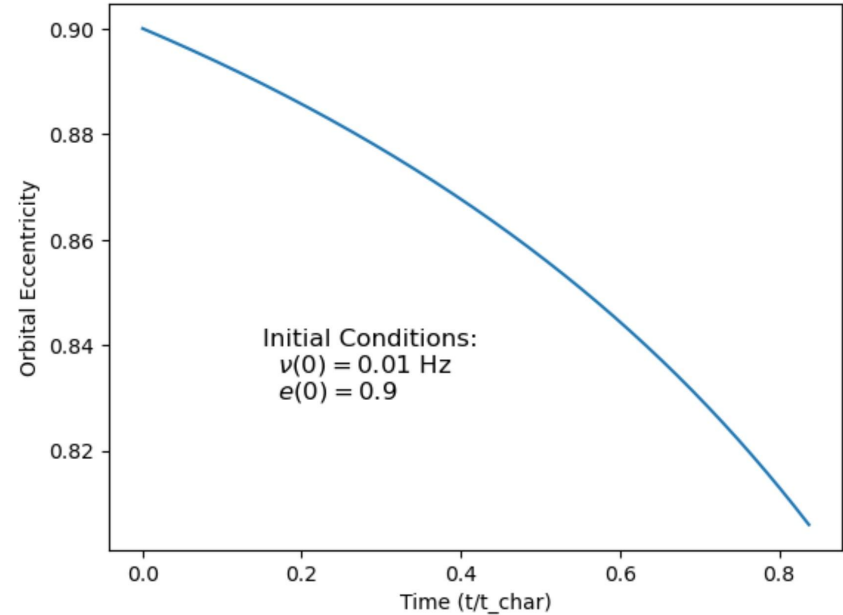
- **End Goal: Calculate the SNR for systems across the orbital frequency and eccentricity parameter space**
- **We want only resolvable systems. Need to filter.**
- **To calculate SNR we need to first calculate the system strain and LISA instrumental noise**

Evolution Trajectory of Systems

5 Year Evolution of Orbital Frequency

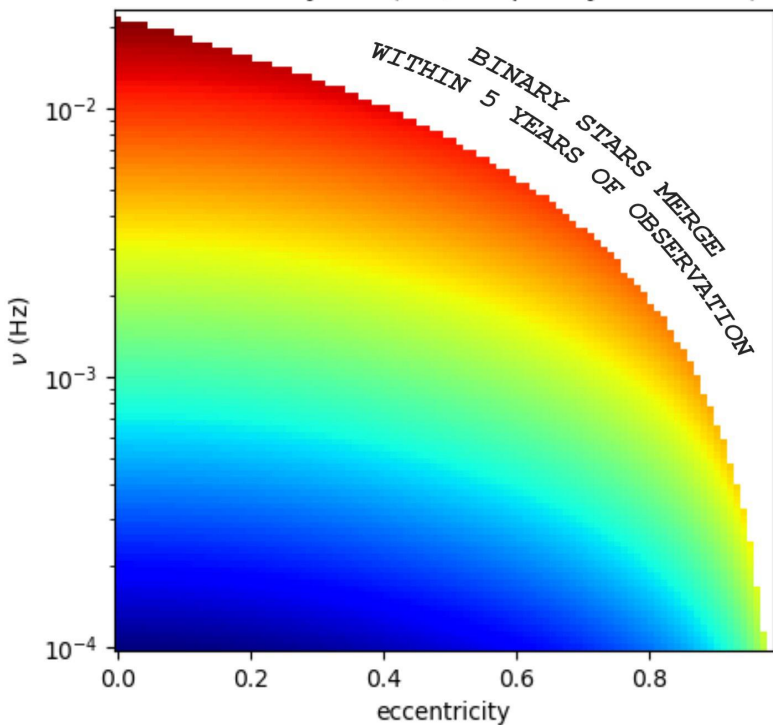


5 Year Evolution of Orbital Eccentricity



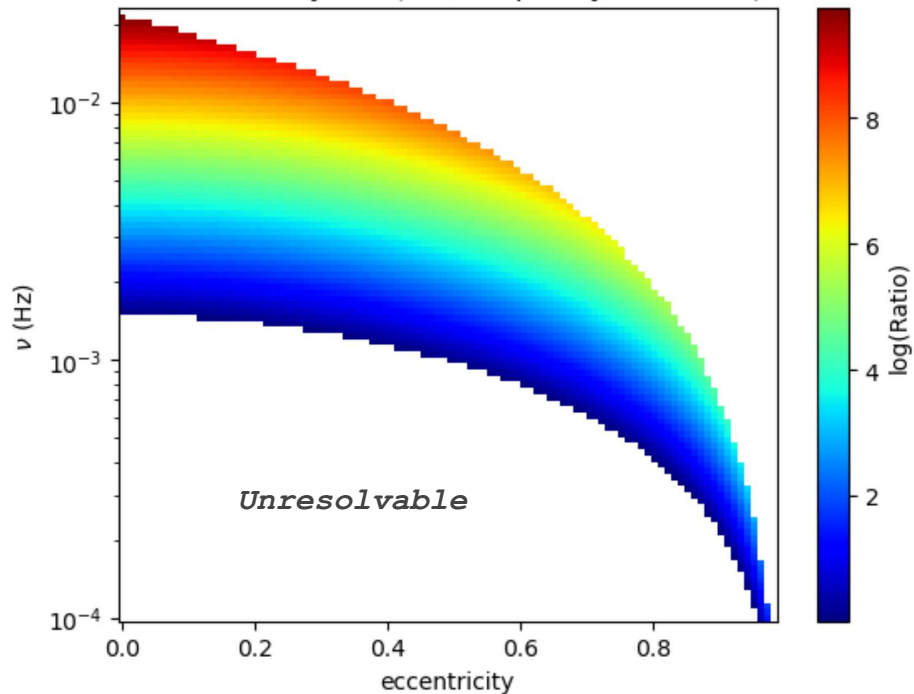
Resolvability

Ratio over 5 years ($\Delta\nu$ / frequency resolution)

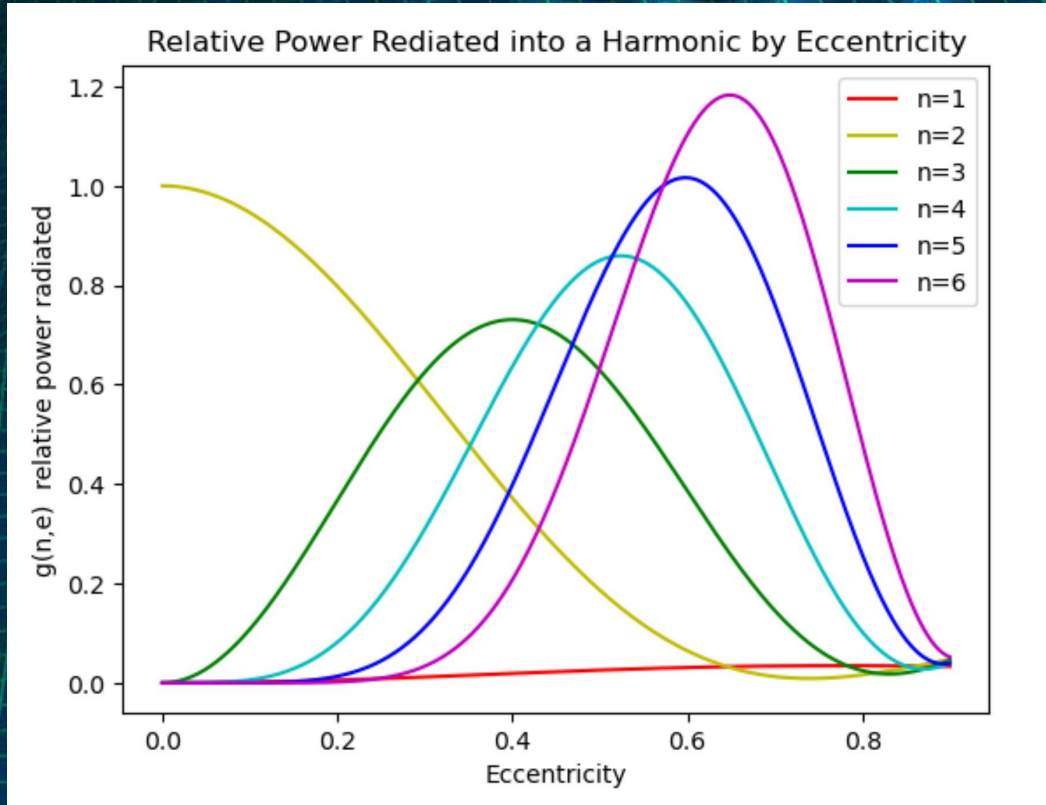


Resolvable Systems Only

Ratio over 5 years ($\Delta\nu$ / frequency resolution)



Calculating $g(n,e)$

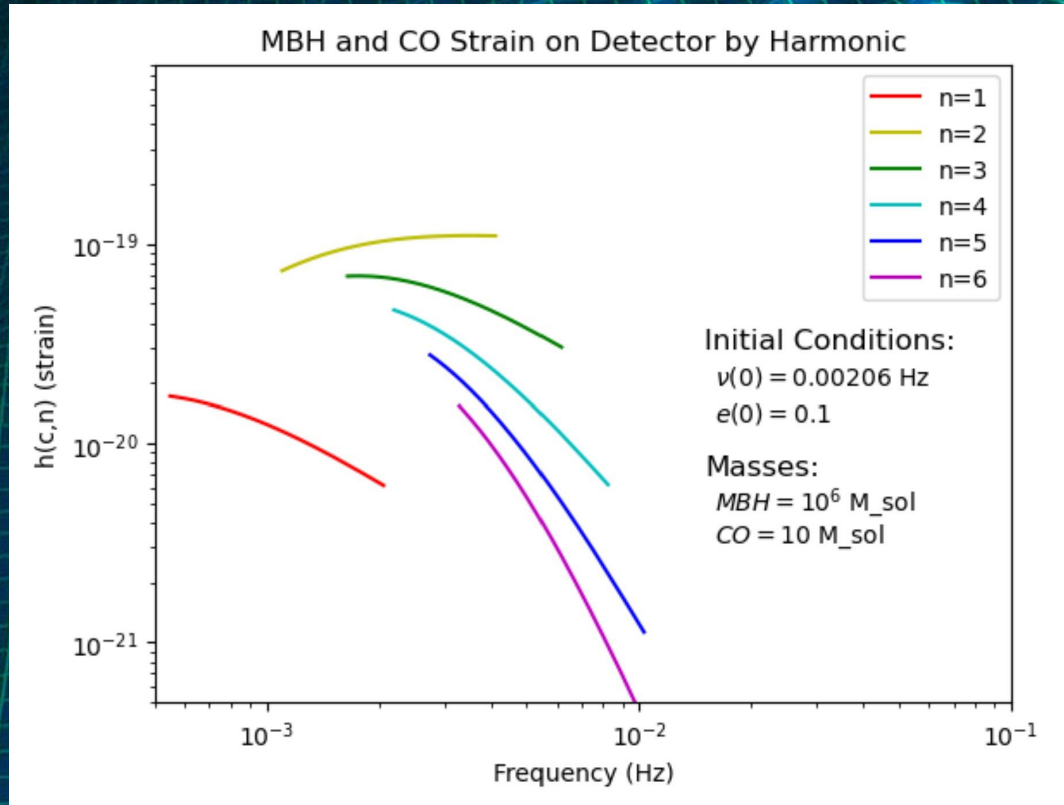


$$\dot{E}_n = \frac{32}{5} \mu^2 M^{4/3} (2\pi\nu)^{10/3} g(n,e)$$

$$g(n,e) = \frac{n^4}{32} \left\{ \left[J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{2}{n}J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne) \right]^2 + (1-e^2)[J_{n-2}(ne) - 2J_n(ne) + J_{n+2}(ne)]^2 + \frac{4}{3n^2}[J_n(ne)]^2 \right\}$$

Barack and Cutler 2004, Eqs. 57 & 58

Calculating $h(c,n)$

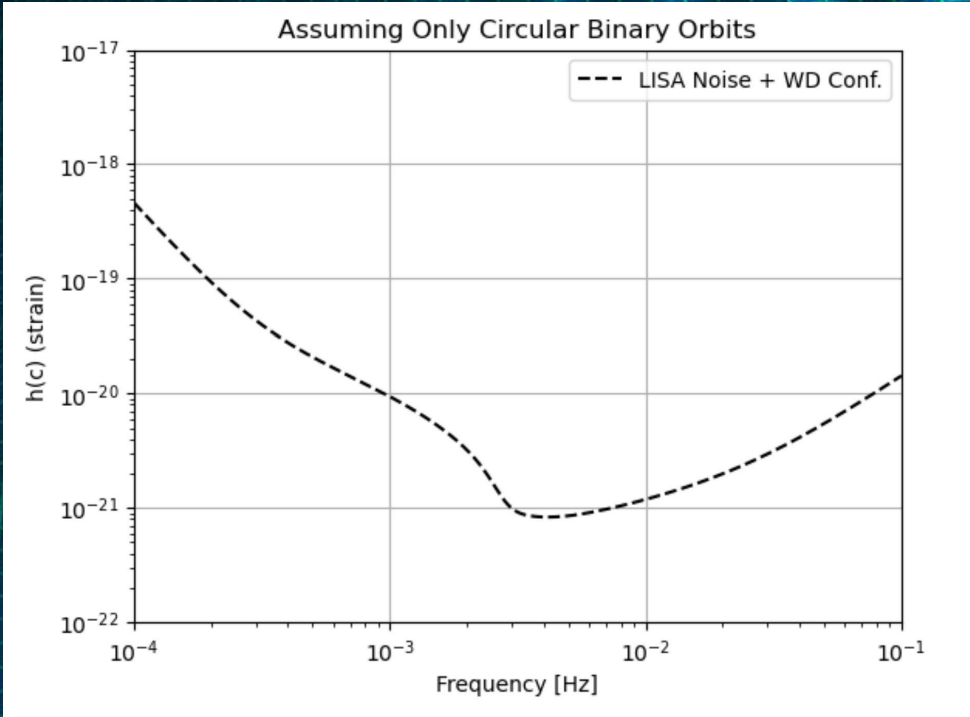


$$h_{c,n} = (\pi D)^{-1} \sqrt{2 \dot{E}_n / \dot{f}_n},$$

$$\dot{E}_n = \frac{32}{5} \mu^2 M^{4/3} (2\pi\nu)^{10/3} g(n,e)$$

Barack and Cutler 2004, Eqs. 56 & 57

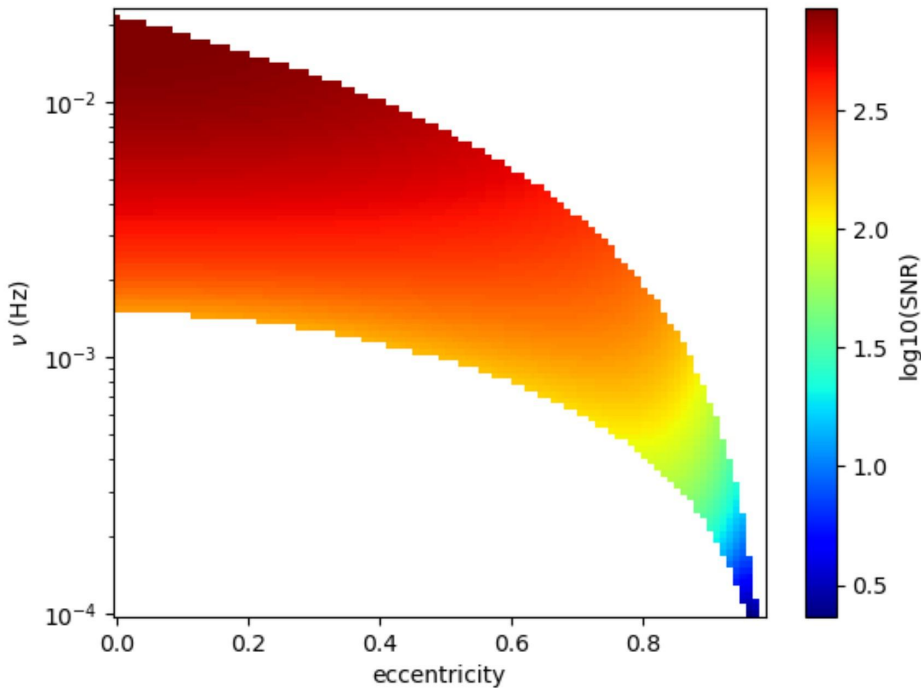
The LISA Noise



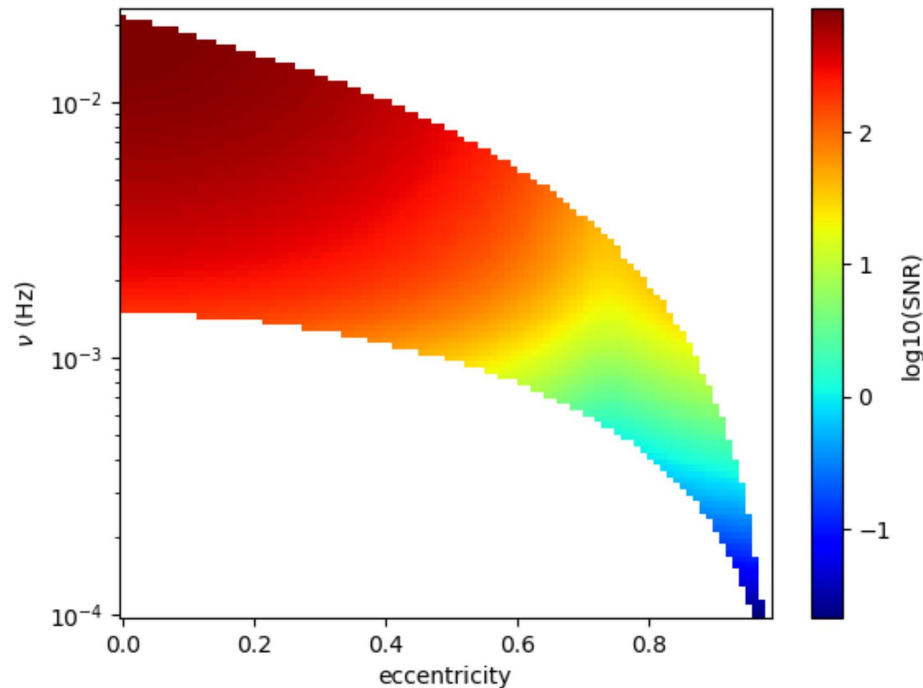
- Instrumental Noise
- WD Confusion Noise
- Only considers circular orbits

SNR Investigation

Total SNR for Harmonics 1-20

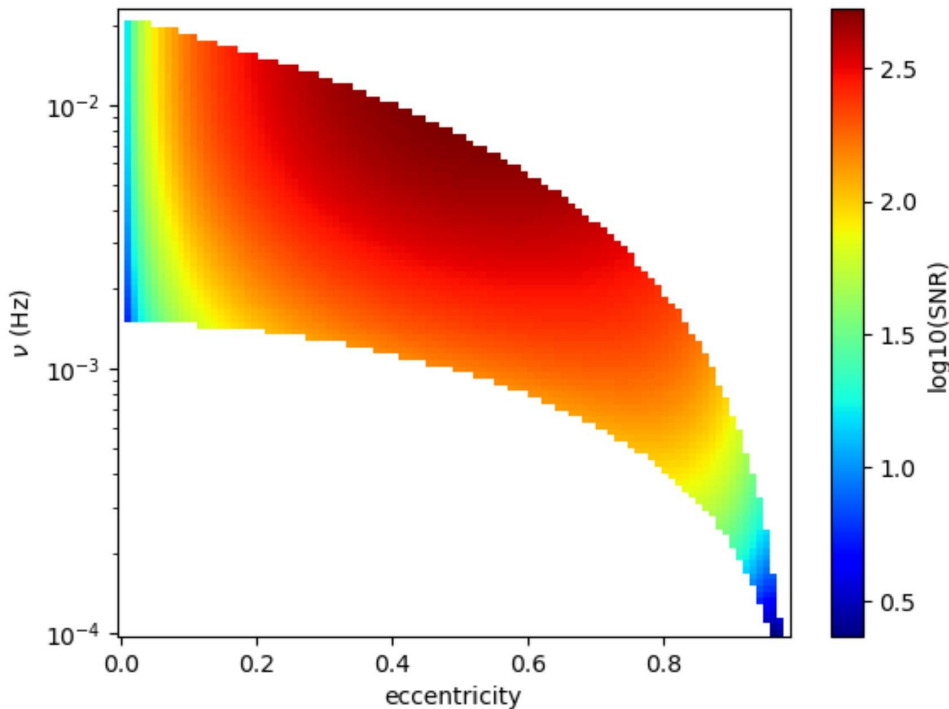


SNR for 2nd Harmonic



SNR Investigation

SNR for All but 2nd Harmonic



- **Total SNR range from ~ 2 to ~ 1000**
- **These Harmonics are the ‘smoking gun’ of eccentricity.**
- **Almost the entire parameter space should be detectable**

References

Barack and Cutler. “LISA Capture Sources: Approximate Waveforms, Signal-to-Noise Ratios, and Parameter Estimation Accuracy.” *Phys.Rev.D* 69

<https://doi.org/10.48550/arXiv.gr-qc/0310125>

Bonetti and Sesana. “Gravitational Wave Background from Extreme Mass Ratio Inspirals.” *Phys.Rev.D* 102

<https://doi.org/10.48550/arXiv.2007.14403>

The ODEs

- **Orbital Frequency**
- **Orbital Eccentricity**
- **Post-Newtonian terms don't contribute significantly**

$$\begin{aligned} \frac{de}{dt} = & \frac{-e}{15} \left(\frac{c^3 \mu}{GM^2} \right) (1 - e^2)^{-\frac{7}{2}} \left(2\pi \frac{GM}{c^3} \nu \right)^{\frac{8}{3}} \left[(304 + 121e^2) (1 - e^2) \left(1 + 12 \left(2\pi \frac{GM}{c^3} \nu \right)^{\frac{2}{3}} \right) \right. \\ & \left. - \frac{1}{56} \left(2\pi \frac{GM}{c^3} \nu \right)^{\frac{2}{3}} ((8)(16705) + (12)(9082)e^2 - 25211e^4) \right] \\ & + e \left(\frac{c^3}{GM} \right) \left(\frac{c^6 S}{G^2 M^2} \right) \cos \lambda \left(2\pi \frac{GM}{c^3} \nu \right)^{\frac{11}{3}} (1 - e^2)^{-4} \left[\left(\frac{1364}{5} \right) + \left(\frac{5032}{15} \right) e^2 + \left(\frac{263}{10} \right) e^4 \right] \end{aligned}$$