

Last time, we considered the vector field

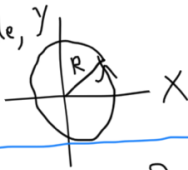
$$\vec{v} = \hat{z} \times \frac{\hat{r}}{r} = \frac{\hat{\phi} \sin\theta}{r} \quad (\text{from HW 1})$$

We found:

$$\nabla \times \vec{v} = \frac{2z\hat{r}}{r^3}$$

This presents a problem when we consider Stokes' theorem in the $z=0$ plane:

(let C be the circle, y
 D the disk)



$$\theta = \frac{\pi}{2}, \text{ so } \sin\theta = 1$$

$$\vec{v} \rightarrow \hat{\phi}/r$$

$$\oint_C \vec{v} \cdot d\vec{l} = \frac{1}{R} 2\pi R = 2\pi \quad (\text{INDEPENDENT OF } R!)$$

BUT $\int_D (\nabla \times \vec{v}) \cdot d\vec{a} = 0$, because $d\vec{a} = \hat{z} da$
 and $\nabla \times \vec{v} = 0$ for $z=0$ (and $\hat{r} \cdot \hat{z} = 0$ FOR $z=0$).

WHAT GIVES?

TRY A DIFFERENT SURFACE, BOUNDED BY THE SAME PATH:

$$\nabla \times \vec{v} = \frac{2\cos\theta \hat{r}}{R^2}$$

$$d\vec{a} = R^2 \sin\theta d\theta d\phi \hat{r}$$

$$(\nabla \times \vec{v}) \cdot d\vec{a} = 2\cos\theta \sin\theta d\theta d\phi$$

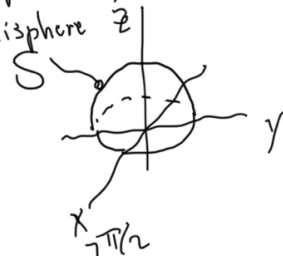
$$\int_S (\nabla \times \vec{v}) \cdot d\vec{a} = \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta 2\cos\theta \sin\theta$$

$$= 2\pi \int_0^{\pi/2} 2\cos\theta \sin\theta d\theta = 2\pi [\sin^2\theta]_0^{\pi/2}$$

$$\therefore \int_S (\nabla \times \vec{v}) \cdot d\vec{a} = 2\pi$$

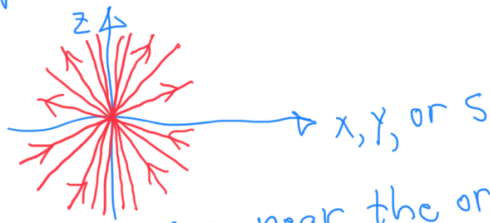
INDEPENDENT OF R
 SAME AS ABOVE!

$r=R, \theta \leq \pi/2$
 hemisphere



Let's draw $\nabla \times \vec{v} = 2z \hat{r} / r^3 = 2 \cos \theta \hat{r} / R^2$:

DISCUSS!



Notice what's happening near the origin!
Unlike anywhere else in the $z=0$ plane,
it looks like the curl could be really big,
and in the \hat{z} direction. Proposal:

$$\nabla \times \vec{v} = \frac{2z \hat{r}}{r^3} + 2\pi \delta(\vec{r}) \hat{z}$$

DISCUSS-