

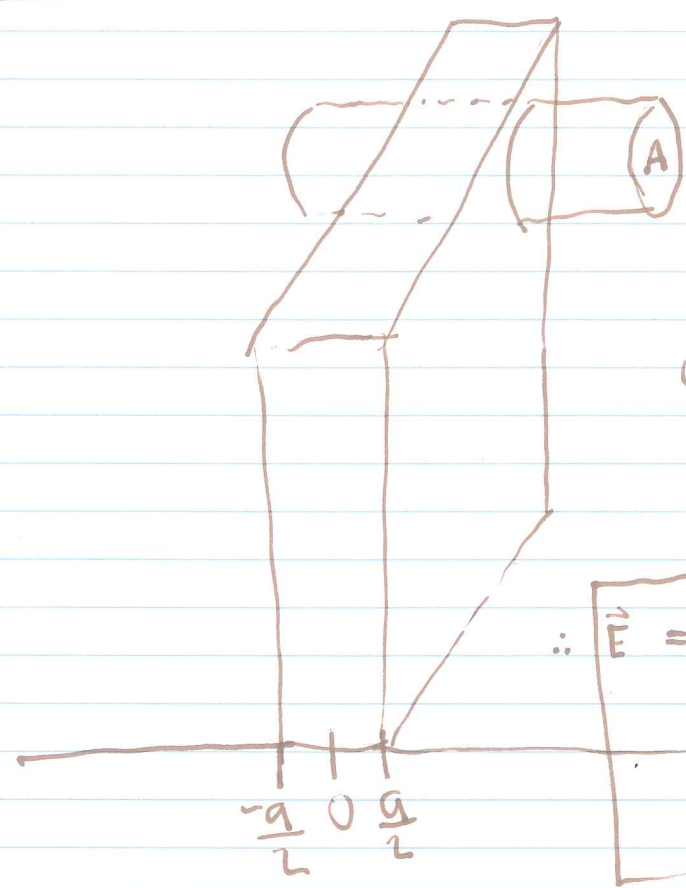
LAST TIME, WE DERIVED GAUSS'S LAW: (§2.2)

$$\nabla \cdot \vec{E} = \rho / \epsilon_0 \quad (\text{DIFFERENTIAL FORM})$$

$$\oint \vec{E} \cdot d\vec{a} = Q_{\text{enc.}} / \epsilon_0 \quad (\text{INTEGRAL FORM})$$

TODAY'S PURPOSE IS TO GET SOME PRACTICE APPLYING GAUSS'S LAW TO PROBLEMS INVOLVING PLANAR, CYLINDRICAL, OR SPHERICAL SYMMETRY.

EXAMPLE FINITE THICKNESS SLAB,  $\rho = \text{CONST.}$



$x > a/2:$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc.}}}{\epsilon_0}$$

$$2EA = \frac{Aa\rho}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{a\rho}{2\epsilon_0} \hat{x}$$

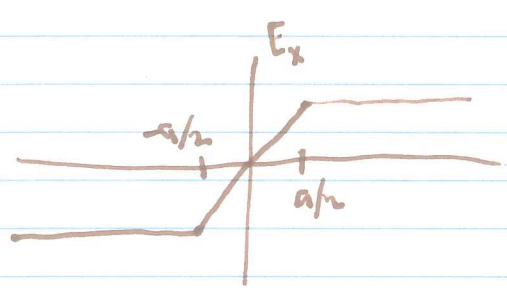
$0 < x < a/2:$

$$2EA = \frac{2Ax\rho}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\rho}{\epsilon_0} x\hat{x}$$

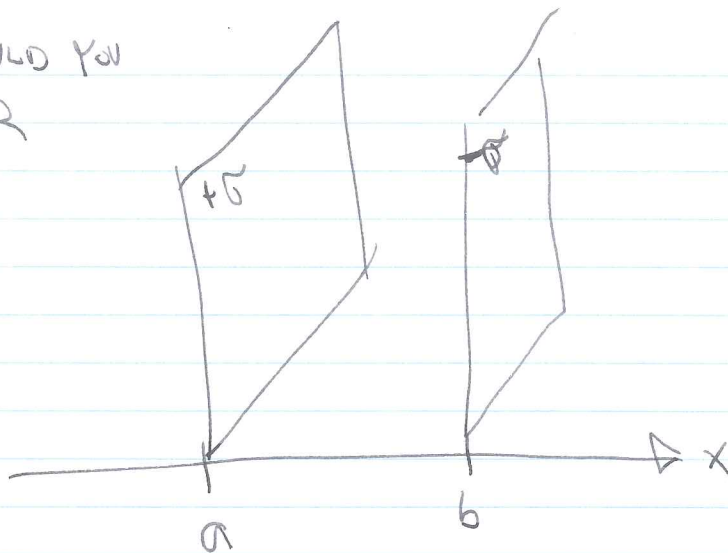
$$\therefore \vec{E} = \begin{cases} -\frac{a\rho}{2\epsilon_0} \hat{x}, & x < -a/2 \\ \frac{\rho}{\epsilon_0} x\hat{x}, & -a/2 < x < a/2 \\ \frac{a\rho}{2\epsilon_0} \hat{x}, & x > a/2 \end{cases}$$

$\triangle x$



## EXERCISE

How would you  
solve for  
 $\vec{E}$ ?



(SUPERPOSITION OF 2 SHEETS OF CHARGE)

OTHER WAY ROUND:

GIVEN  $\vec{E} = E_0 \hat{x}$ , WHAT IS  $\rho$ ?

$$\nabla \cdot \vec{E} = 0 \quad ??$$

ALSO  $\nabla \times \vec{E} = 0$ , SO THIS IS CERTAINLY A VALID FIELD.

OF COURSE, IF YOU HAVE

$$\vec{E} = \begin{cases} 0, & x < a; \\ E_0 \hat{x}, & a < x < b; \\ 0, & x > b. \end{cases}$$

THEN THE CHARGE DENSITY IS OBVIOUS.

TECHNICALLY, ANY ~~ANY~~ FIELD  $\vec{E}' \ni \nabla \times \vec{E}' = \nabla \cdot \vec{E}' = 0$   
CAN BE ADDED TO A SOLUTION  $\vec{E}$  AND STILL SATISFY  
GAUSS'S LAW. DOES THAT MEAN COULOMB'S LAW WOULD  
BE SATISFIED?  $\rightarrow$  NO. AND WE CAN FIX THE  
AMBIGUITY BY LOOKING AT BOUNDARIES AND/OR LIMITS,  
E.G.  $\vec{E} \rightarrow 0$  AS  $r \rightarrow \infty$  ...

EXAMPLE  $\rho = \rho_0 e^{-s^2/2a^2}$ , FIND  $\vec{E}$  & SKETCH  $E(s)$



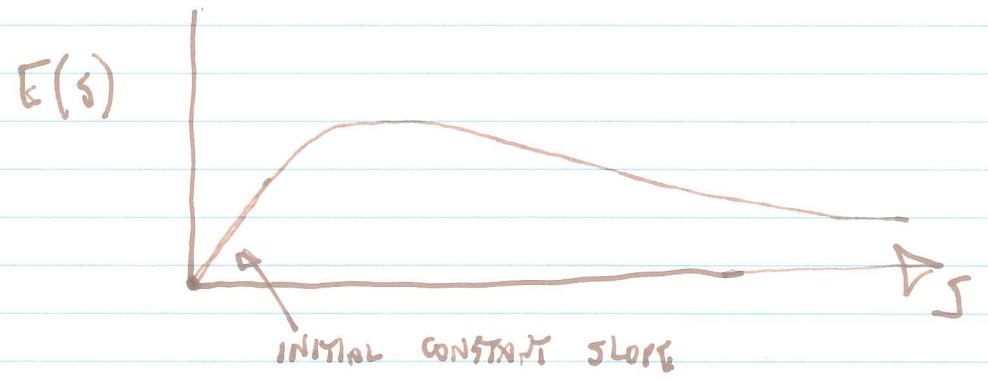
$$\oint \vec{E} \cdot d\vec{a} = 2\pi l E(s)$$

$$Q_{enc} = \rho_0 l \int_0^a e^{-s'^2/2a^2} 2\pi s' ds'$$

$$= -2\pi \rho_0 l a^2 e^{-s'^2/2a^2} \Big|_0^a$$

$$= -2\pi \rho_0 l a^2 [e^{-s^2/2a^2} - 1]$$

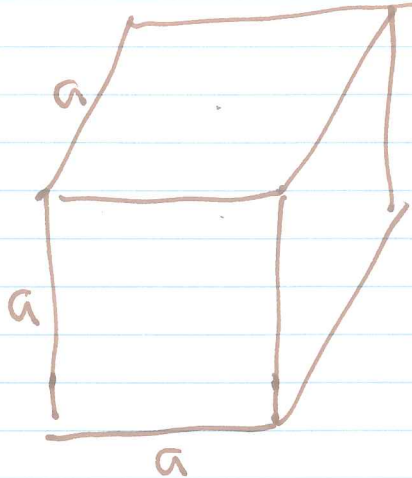
$$E(s) = \frac{\rho_0 a^2}{\epsilon_0 s} (1 - e^{-s^2/2a^2})$$



DISCUSS  $s \rightarrow \infty$ . WHAT IS THE EFFECTIVE LINEAR CHARGE  $\lambda$ ?

EXAMPLE / EXERCISE

CAN WE USE GAUSS'S LAW TO SOLVE THE  
CUBE OF CONSTANT  $\rho$  ?

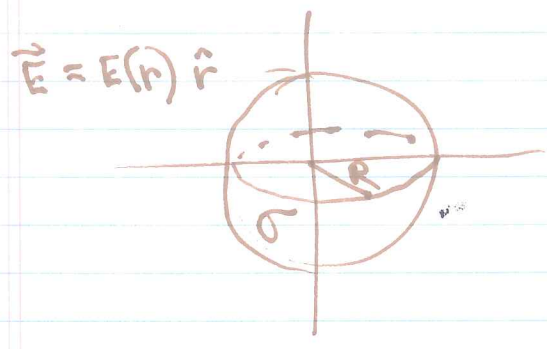


WHY OR  
WHY NOT ?

→ CAN YOU IDENTIFY A SURFACE SUCH THAT  
 $\vec{E} \cdot \hat{n} = |E|$  ?

EXERCISE / EXAMPLE

FIND  $\vec{E}$  FOR A SPHERICAL SHELL WITH CONSTANT SURFACE CHARGE DENSITY  $\sigma$ .



$r > R$ :

$$\oint \vec{E} \cdot d\vec{a} = 4\pi r^2 E(r)$$

$$Q_{ENC} = 4\pi R^2 \sigma$$

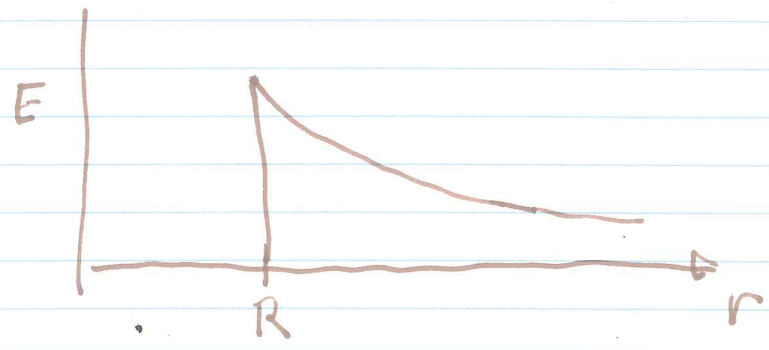
$$\therefore E(r) = \frac{\sigma}{\epsilon_0} \frac{R^2}{r^2}$$

$r < R$ :  $Q_{ENC} = 0$

$$\therefore E(r) = 0$$

$$\vec{E} = \begin{cases} \frac{R^2 \sigma \hat{r}}{r^2 \epsilon_0}, & r > R \\ 0, & r < R \end{cases}$$

USING  $Q = 4\pi R^2 \sigma$ ,  
THIS BECOMES  
 $\frac{Q \hat{r}}{4\pi \epsilon_0 r^2}$ .

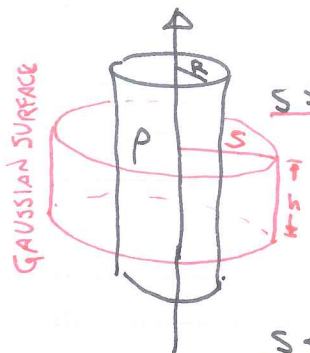


CAN THIS BE VIEWED AS AN INFINITE SLAB IF WE ZOOM IN NEAR THE SURFACE OF THE SPHERE?

DISCUSS.

## CREATIVELY COMBINING GAUSS'S LAW WITH SUPERPOSITION

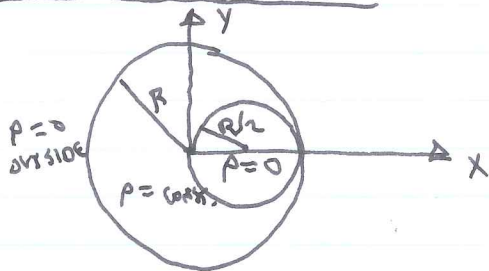
SOLID CYLINDER: IF  $\rho = \text{const.}$ ,  $s < R$ , THEN THE FIELD IS...  $\vec{E} = E(s) \hat{s}$



$s > R$ :  $\oint \vec{E} \cdot d\vec{A} = 2\pi s h E(s)$  }  $2\pi s h E = \pi R^2 h \rho / \epsilon_0$   
 $Q_{\text{enc}} = \pi R^2 h \rho$   
 $\therefore \vec{E}(s) = \frac{R^2 \rho}{2\epsilon_0 s} \hat{s}, \quad s > R.$

$s < R$ : THE ONLY DIFFERENCE WITH THIS CASE IS  
 $Q_{\text{enc}} = \pi s^2 h \rho \quad (R^2 \rightarrow s^2).$  SO,  
 $\vec{E}(s) = \frac{\rho s}{2\epsilon_0} \hat{s}, \quad s < R.$

### COMBINATION CYLINDER:



WE CAN VIEW THE "DRILLED" CYLINDER AS COMBINING A CYLINDER OF CONSTANT CHARGE DENSITY  $\rho$  WITH ~~THE~~ A SMALLER CYLINDER OF CHARGE DENSITY  $-\rho$ :

$$\vec{E}_{\text{big}} = \begin{cases} \frac{R^2 \rho}{2\epsilon_0 s} \hat{s}, & s > R \\ \frac{\rho s}{2\epsilon_0} \hat{s}, & s < R \end{cases}$$

$$\vec{s}_{\text{small}} = \left(x - \frac{R}{2}\right) \hat{x} + y \hat{y}$$

$$s_{\text{small}} = \sqrt{\left(x - \frac{R}{2}\right)^2 + y^2}$$

$$\vec{E}_{\text{small}} = \begin{cases} -\frac{R^2 \rho}{8\epsilon_0} \frac{\left[\left(x - \frac{R}{2}\right) \hat{x} + y \hat{y}\right]}{\left[\left(x - \frac{R}{2}\right)^2 + y^2\right]}, & \left(x - \frac{R}{2}\right)^2 + y^2 < \frac{R^2}{4} \\ -\frac{\rho}{2\epsilon_0} \left[\left(x - \frac{R}{2}\right) \hat{x} + y \hat{y}\right], & \left(x - \frac{R}{2}\right)^2 + y^2 > \frac{R^2}{4} \end{cases}$$

$$\vec{E}_{\text{TOT}} = \vec{E}_{\text{big}} + \vec{E}_{\text{small}}$$