

Example

Given:

$$\vec{E} = \begin{cases} A \frac{\hat{r}}{r^2}, & a < r < b \quad (A \text{ a constant}) \\ 0, & \text{elsewhere} \end{cases}$$

Find the charge distribution & $V(\vec{r})$.

Consider a spherical Gaussian surface of radius r .

$$Q_{\text{ENC}} = \epsilon_0 \oint \vec{E} \cdot d\vec{a} = \begin{cases} 4\pi\epsilon_0 A, & a < r < b \\ 0, & r < a \text{ or } r > b \end{cases}$$

Interpretation: Two concentric spherical shells,



$$V = - \int_{\vec{r}} \vec{E} \cdot d\vec{l} = \int_r^{\infty} E_r dr$$
$$a < r < b: V = \int_r^b E_r dr = \left(\frac{A}{r} - \frac{A}{b} \right)$$

$$\therefore V = \begin{cases} \frac{A}{a} - \frac{A}{b}, & r \leq a \\ \frac{A}{r} - \frac{A}{b}, & a < r < b \\ 0, & r \geq b \end{cases}$$

