

REVIEW - \vec{D} , \vec{E} , \vec{P}

~~EXAMPLE~~

$$\nabla \cdot \vec{E} = \rho / \epsilon_0,$$

$$\nabla \times \vec{E} = 0,$$

$$\nabla \cdot \vec{P} = -\rho_b,$$

$$\rho = \rho_b + \rho_f,$$

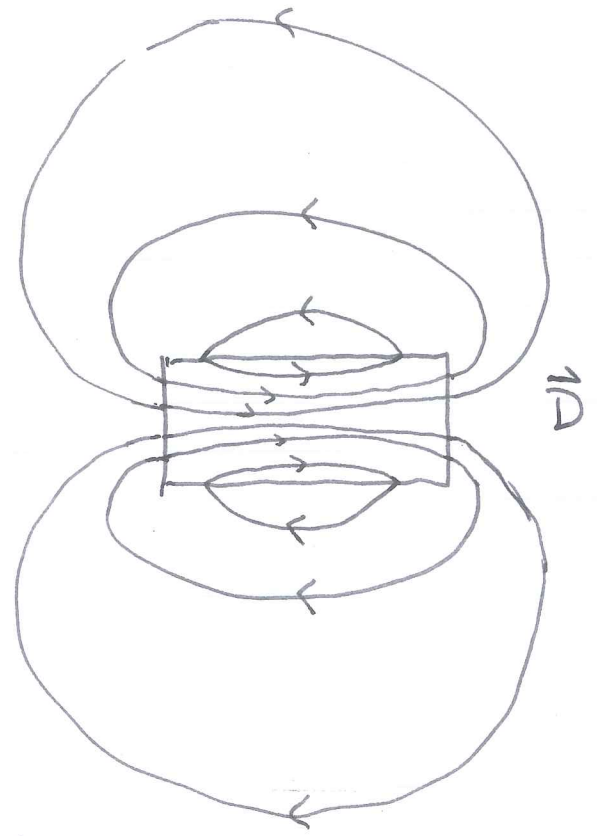
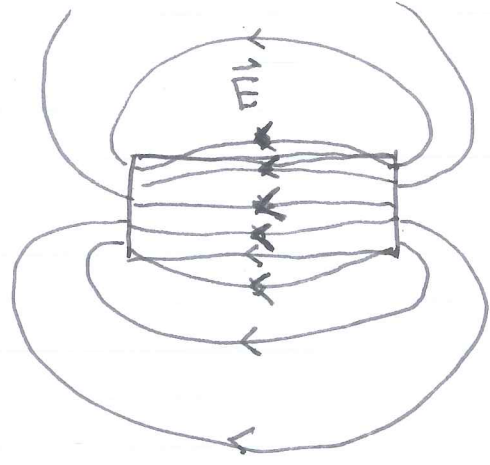
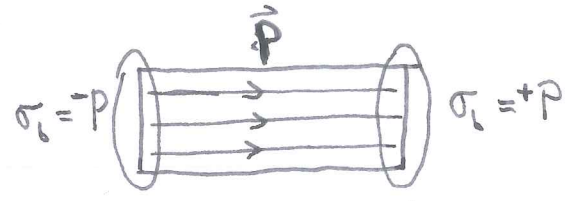
$$\text{SINCE } \vec{D} = \epsilon_0 \vec{E} + \vec{P},$$

$$\nabla \cdot \vec{D} = \rho_f ; \quad \nabla \times \vec{D} = \nabla \times \vec{P}.$$

AND FROM THIS COME ALL THE BOUNDARY CONDITIONS (GRIFFITHS EQS. 4.26-4.29).

EXAMPLE (GRIFFITHS PROB. 4.17)

FIELDS OF A BAR ELECTRET



NOTE THAT $\nabla \cdot \vec{D} = 0$ SINCE $\rho_f = 0$
 \vec{D} OUTSIDE $\propto \vec{E}_{out}$ OUTSIDE.

GRIFFITHS 4.4.1 LINEAR DIELECTRICS

OFTEN, POLARIZATION IS CAUSED BY AN APPLIED FIELD. ~~THE~~ THE SIMPLEST EXAMPLE (AND THE TOPIC OF THE REST OF THIS CHAPTER) IS A DIRECTLY PROPORTIONAL RESPONSE,

$$\vec{P} \propto \vec{E}$$

NOTE THAT $\epsilon_0 \vec{E}$ HAS THE SAME UNITS AS \vec{P} . SO IT MAKES SENSE TO DEFINE A DIMENSIONLESS ELECTRIC SUSCEPTIBILITY:

$$\vec{P} = \chi_e \epsilon_0 \vec{E} \quad (\text{formerly } \chi_e \epsilon_0 \vec{E})$$

NOW, ~~THE~~ THE DISPLACEMENT IS:

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = (1 + \chi_e) \epsilon_0 \vec{E}$$

OR

$$\vec{D} = \epsilon \vec{E}, \text{ WHERE } \epsilon = \epsilon_0 (1 + \chi_e) \text{ IS THE PERMITTIVITY.}$$

WE ALSO COMMONLY DEFINE

$$\epsilon_r \equiv 1 + \chi_e = \frac{\epsilon}{\epsilon_0}, \text{ THE DIELECTRIC CONSTANT. (OR RELATIVE PERMITTIVITY)}$$

NOTE THAT χ_e AND ϵ_r ARE BOTH DIMENSIONLESS.

$$\star \text{ IN VACUUM, } \epsilon = \epsilon_0 \text{ (} \chi_e = 0, \epsilon_r = 1 \text{)} \star$$

Q: WHAT WOULD YOU EXPECT IS THE SIGN OF χ_e ?
WHAT DOES THIS IMPLY FOR ϵ AND ϵ_r ?

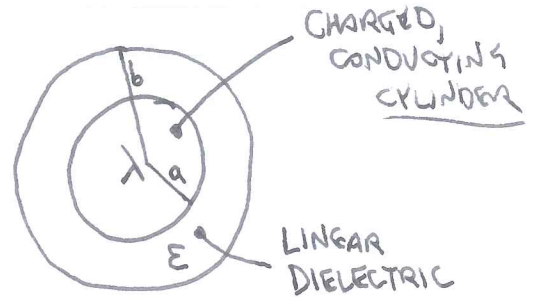
$$A: \chi_e \geq 0; \epsilon \geq \epsilon_0; \epsilon_r \geq 1$$

EXAMPLE

FIND \vec{E} , \vec{P} , \vec{D} , BOUND CHARGES ...

Q: WHICH IS EASIEST?

- A: • \vec{P} REQUIRES KNOWING \vec{E}
- \vec{E} REQUIRES KNOWING THE BOUND CHARGES, OR BOTH \vec{P} AND \vec{D} .



• WHAT ABOUT \vec{D} ?

→ SINCE THE GEOMETRY IS CYLINDRICALLY SYMMETRIC, WITH RADIAL FIELDS, ASSUME $\nabla \times \vec{D} = \nabla \times \vec{P} = 0$ AND USE $\nabla \cdot \vec{D} = \rho_f$.

THE ONLY FREE CHARGE IS λ , SO ~~THE~~

$$\oint \vec{D} \cdot d\vec{a} = (2\pi s l) D = \lambda l$$

$$\Rightarrow \vec{D} = \frac{\lambda \hat{s}}{2\pi s}, \quad r > a$$

NOW $\vec{D} = \epsilon \vec{E}$, SO:

$$\vec{E} = \begin{cases} 0, & r < a \text{ (CONDUCTOR)} \\ \frac{\lambda \hat{s}}{2\pi \epsilon s}, & a < r < b \\ \frac{\lambda \hat{s}}{2\pi \epsilon_0 s}, & r > b \end{cases}$$

$$\text{AT } \underline{r=a}, \quad \sigma = \epsilon_0 (E_{out}^\perp - E_{in}^\perp) = \frac{\lambda \epsilon_0}{2\pi \epsilon a} = \frac{\lambda}{2\pi a} \left(\frac{\epsilon_0}{\epsilon} \right)$$

$$\sigma_f = \frac{\lambda}{2\pi a} \text{ (GEOMETRY!)}$$

$$\text{SO } \sigma_b = \sigma - \sigma_f = \frac{\lambda}{2\pi a} \left(\frac{\epsilon_0}{\epsilon} - 1 \right)$$

$$\therefore \sigma_b = \frac{-\lambda}{2\pi a} \left(1 - \frac{\epsilon_0}{\epsilon} \right)$$

↑
WHY DID I REWRITE IT THIS WAY?

Q: WHAT IS \vec{P} ?
WHAT IS σ_b @ $r=b$?

§ 4.4.2 — BOUNDARY VALUE PROBLEMS IN LINEAR DIELECTRICS

IN A LINEAR DIELECTRIC,

$$\rho_f = \nabla \cdot \vec{D} = \nabla \cdot (\epsilon \vec{E}) = \epsilon \nabla \cdot \vec{E} + \vec{E} \cdot \nabla \epsilon$$

PROVIDED $\epsilon = \text{CONST.}$,

$$\boxed{-\nabla^2 V = \nabla \cdot \vec{E} = \rho_f / \epsilon.}$$

IF THERE IS NO FREE CHARGE, WE HAVE LAPLACE'S EQUATION:

$$\nabla^2 V = 0. \quad (\text{WHERE } \rho_f = 0)$$

THE BOUNDARY CONDITIONS ARE:

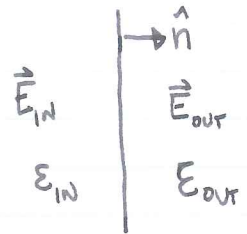
$$\epsilon_{\text{out}} E_{\text{out}}^\perp - \epsilon_{\text{in}} E_{\text{in}}^\perp = \sigma_f$$

$$\text{OR: } -\epsilon_{\text{out}} \frac{\partial V_{\text{out}}}{\partial n} + \epsilon_{\text{in}} \frac{\partial V_{\text{in}}}{\partial n} = \sigma_f$$

AND OF COURSE

$$\vec{E}_{\text{out}}^\parallel = \vec{E}_{\text{in}}^\parallel$$

$$V_{\text{out}} = V_{\text{in}}$$



NOW, WHAT CAN WE DO WITH THIS?

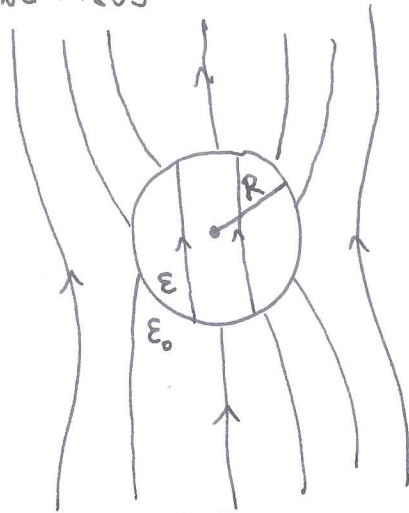
Q: WHAT HAPPENS IF A FREE CHARGE q (PERHAPS AN ELECTRON) LANDS IN THE BULK OF AN INSULATOR?

A: IT ACTS LIKE q/ϵ_r .

Q: WHAT IS THE LIMIT $\epsilon \rightarrow \infty$?

A: DIELECTRIC \rightarrow CONDUCTOR.

EXAMPLE (GRIFFITHS 4.7)
DIELECTRIC SPHERE IN A UNIFORM EXTERNAL FIELD



- I. As $r \rightarrow \infty$, $\vec{E} \rightarrow E_0 \hat{z}$
 $V \rightarrow \vec{E}_0 \cdot \vec{z} = -E_0 r \cos \theta$
- II. $V_{out} = V_{in}$ @ $r = R$
- III. $\epsilon_0 \frac{\partial V_{out}}{\partial r} \Big|_R = \epsilon \frac{\partial V_{in}}{\partial r} \Big|_R$

SINCE THE GENERAL SOLUTION IS

$$V(r, \theta) = \sum_l (A_l r^l + B_l r^{-(l+1)}) P_l(\cos \theta),$$

AND $P_1 = \cos \theta$, B.C. (I) SUGGESTS TO LOOK AT JUST $l = 1$:

$$V_{in} = (A_1 r + B_1 r^{-2}) \cos \theta$$

$$V_{out} = (C_1 r + D_1 r^{-2}) \cos \theta$$

FROM (I), $C_1 = -E_0$.

FROM (II), $A_1 R = -E_0 R + D_1 R^{-2} \Rightarrow A_1 = -E_0 + D_1/R^3$

AND (III): $\frac{\partial V_{out}}{\partial r} \Big|_R = (C_1 - 2D_1/R^3) \cos \theta = -(E_0 + 2D_1/R^3) \cos \theta$

$$\frac{\partial V_{in}}{\partial r} \Big|_R = (A_1) \cos \theta$$

$$\text{SO } -E_0 (E_0 + 2D_1/R^3) = \epsilon A_1 = -\epsilon E_0 + \epsilon D_1/R^3$$

$$\Rightarrow (\epsilon - \epsilon_0) E_0 = (\epsilon + 2\epsilon_0) D_1/R^3$$

$$\Rightarrow D_1 = E_0 R^3 \left(\frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0} \right), \quad A_1 = E_0 \frac{-3\epsilon_0}{\epsilon + 2\epsilon_0}$$

UNIFORM FIELD INSIDE!

$$\therefore V_{in} = \frac{-3\epsilon_0 \epsilon_0}{\epsilon + 2\epsilon_0} r \cos \theta, \quad V_{out} = \left[-E_0 r + E_0 \frac{R^3}{r^2} \left(\frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0} \right) \right] \cos \theta$$

Q: WHAT DOES THE LIMIT $\epsilon \rightarrow \infty$ REPRESENT?
A: A CONDUCTING SPHERE.