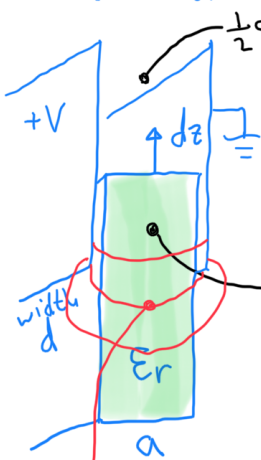


# Force on a linear dielectric § 4.4.4



How much does the energy change if the dielectric moves upward by  $dz$ ?

General Idea:  
Ignore fringing fields near edges of

$$\frac{1}{2} \epsilon_r \epsilon_0 E^2; \quad E = V_0/a.$$

$$W = \frac{1}{2} \epsilon \int E^2 d\tau$$

$$dW = \alpha d \cdot dz \left( \frac{1}{2} E^2 \right) (\epsilon - \epsilon_0)$$

$$= \frac{1}{2} E^2 \alpha d (\epsilon - \epsilon_0) dz$$

$$\text{Now, } dW = F dz + V_0 dQ$$

$$dQ = V_0 dC;$$

$$\text{Recall } C = \frac{\epsilon A}{a}, \text{ so}$$

$$dC = \frac{(\epsilon - \epsilon_0) A}{a} = \frac{(\epsilon - \epsilon_0) d dz}{a}$$

$$\text{So, } V_0 dQ = V_0^2 (\epsilon - \epsilon_0) \frac{d}{a} dz = E^2 (\epsilon - \epsilon_0) \alpha d dz$$

Putting it all together,

$$F = \frac{dW}{dz} - V_0 \frac{dQ}{dz}$$

$$F = -\frac{1}{2} E^2 \alpha d (\epsilon - \epsilon_0) = -\frac{1}{2} V_0^2 \frac{d}{a} (\epsilon - \epsilon_0)$$

Notice that the force is negative, which means the dielectric is "sucked into" the capacitor. Interestingly, the result is equal & opposite to  $dW/dz$ !

The fringing field is where the force is really exerted. The dipoles in the dielectric are attracted to the stronger field region between the plates.

This is the force required to push the dielectric into the capacitor.

$\epsilon_0 \chi_e$