

Recall force on a dipole \vec{p} :

$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$$

What is the force per unit volume on a dielectric?

$$\frac{d\vec{F}}{d\tau} = (\vec{P} \cdot \nabla) \vec{E}$$

In a linear dielectric, $\vec{P} = \epsilon_0 \chi_e \vec{E}$; so

$$\frac{d\vec{F}}{d\tau} = \epsilon_0 \chi_e (\vec{E} \cdot \nabla) \vec{E}$$

Now consider the identity:

$$\nabla(\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) + (\vec{A} \cdot \nabla) \vec{B} + (\vec{B} \cdot \nabla) \vec{A}.$$

Since $\nabla \times \vec{E} = 0$,

$$(\vec{E} \cdot \nabla) \vec{E} = \frac{1}{2} \nabla(E^2). \text{ so,}$$

$$\frac{d\vec{F}}{d\tau} = \frac{1}{2} \epsilon_0 \chi_e \nabla(E^2).$$

Can the above result be applied to the problem of § 4.4.4 ?!