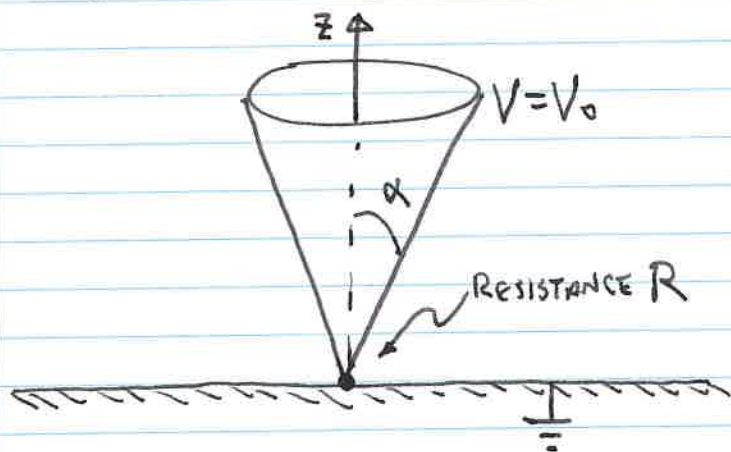


REMEMBER THIS EXAMPLE FROM ELECTROSTATICS?



FIND $\vec{B}(\vec{r})$
FOR $\alpha < \theta < \pi/2$.

EXAMPLE

• SUPPOSE $\vec{J} = K \hat{y} S(x)$. FIND \vec{B} USING AMPERE'S LAW.

- TRY ASSUMING SYMMETRY ABOUT $x=0$.
- TRY ASSUMING $B \rightarrow 0$ AS $x \rightarrow \infty$ (OR $-\infty$?)

• HOW IN GENERAL WOULD YOU FIND \vec{B} FROM $\vec{J} = \hat{y} J(x)$?

• BEGIN WITH ASSUMPTION $\vec{B} = \hat{z} B(x)$.

• DISCUSS HW: $\vec{J} = J_0 \hat{y} e^{-bx}$

• THIS ONLY MAKES SENSE IF WE ASSUME $B \rightarrow 0$ AS $x \rightarrow \infty$.

• IS THIS CONSISTENT WITH THE BIOT-SAVART LAW?

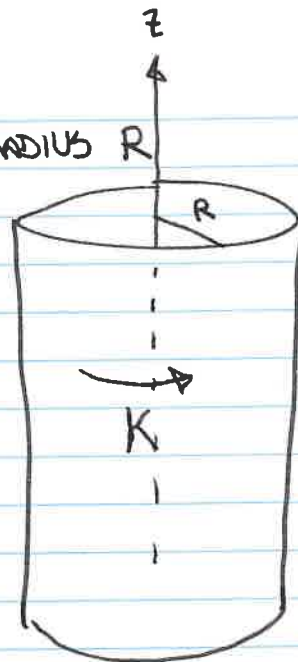
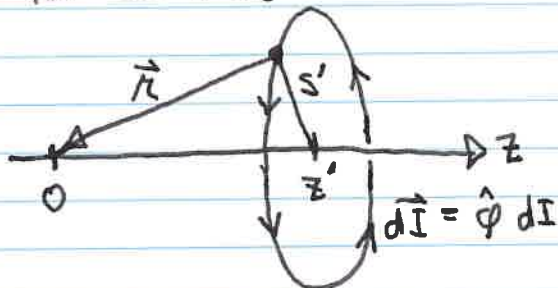
• AT FIRST GLANCE, NO.

• MAYBE IF THERE ARE OTHER, UNSEEN CURRENTS OUTSIDE THE REGION WE ARE CONSIDERING.

BIOT-SAVART LAW EXAMPLE

FIND \vec{B} AT THE CENTER OF A ^(INFINITE) SOLENOID WITH RADIUS R AND SURFACE CURRENT K . *

FIRST, CONSIDER A CIRCULAR LOOP OF WIRE AT CENTERED AT z' :



BIOT-SAVART LAW TO FIND \vec{B} AT THE ORIGIN:

$$d\vec{B} = \frac{\mu_0}{4\pi} \int \frac{d\vec{I}(\vec{r}') \times \vec{r}}{r^3} dl'$$

$$dl' = s' d\phi'; \quad \vec{r} = \vec{r} - \vec{r}' = -\vec{r}'; \quad r = \sqrt{z'^2 + s'^2}$$

$$d\vec{I} \times \vec{r} = dI \hat{\phi} \times (-s' \hat{s} - z' \hat{z}) = dI (+s' \hat{z} - z' \hat{s}) \quad **$$

BY SYMMETRY, ONLY THE \hat{z} TERM CONTRIBUTES TO THE TOTAL. (DISCUSS)

$$d\vec{B} = \frac{\mu_0}{4\pi} \int_0^{2\pi} \underbrace{\frac{dI s' \hat{z}}{(z'^2 + s'^2)^{3/2}}}_{\text{CONSTANTS!}} s' d\phi' = \frac{\mu_0 dI s'^2 \hat{z}}{2(z'^2 + s'^2)^{3/2}} \quad *$$

NOW, FOR THE SOLENOID: $s' = R$, $dI = K dz'$

$$\vec{B} = \int_{z'=-\infty}^{\infty} \frac{\mu_0 K R^2 \hat{z} dz'}{2(z'^2 + R^2)^{3/2}}$$

CONSIDER: $\frac{d}{dx} \frac{x}{\sqrt{x^2 + a^2}} = \frac{1}{\sqrt{x^2 + a^2}} - \frac{1}{2} \frac{2x^2}{(x^2 + a^2)^{3/2}} = \frac{a^2}{(x^2 + a^2)^{3/2}}$

$$\therefore \vec{B} = \frac{\mu_0 K R^2 \hat{z}}{2} \left. \frac{z'}{\sqrt{z'^2 + R^2}} \right|_{-\infty}^{\infty} = \mu_0 K \hat{z}$$

* ~~GRIFFITHS~~ GRIFFITHS EX. 5.9 (USING AMPERE'S LAW) REQUIRED THE ASSUMPTION THAT $B \rightarrow 0$ AS $r \rightarrow \infty$. HERE WE TEST THAT ASSUMPTION!

** DISCUSS CROSS PRODUCTS USING COMPONENT FORM — USUALLY THEY CAN BE DONE QUICKLY.

* COULD BE USED TO FIND THE ON-AXIS FIELD FROM ANY $\vec{J}(s, z) = J(s, z) \hat{\phi}$

EXAMPLE: MAGNETIC "PRESSURE" IN A SOLENOID

WE KNOW THAT A SOLENOID WITH CURRENT DENSITY K_0 HAS A MAGNETIC FIELD $B_0 = \mu_0 K_0$ INSIDE. BUT K_0 STRADDLES A BOUNDARY BETWEEN $B = B_0$ AND $B = 0$. SO WHAT IS THE LORENTZ FORCE (PER UNIT AREA) ON THE WIRES CARRYING K_0 ?

LET THE CURRENT DISTRIBUTION HAVE THICKNESS δ :

$$\vec{J} = \hat{\phi} J(s), \quad a < s < a + \delta; \quad d\vec{K} = \vec{J} ds; \quad \int_a^{a+\delta} J(s) ds = K_0$$

$$\vec{B}(s) = \mu_0 \hat{z} \int_s^{a+\delta} J(s') ds', \quad a < s < a + \delta; \quad \vec{B}|_{s=a} = B_0 \hat{z} = \mu_0 K_0 \hat{z}$$

FORCE PER UNIT VOLUME:

$$\vec{J} \times \vec{B} = \hat{s} J B = \mu_0 \hat{s} \int_s^{a+\delta} J(s') ds'$$

PRESSURE:

$$\vec{P} = \int_a^{a+\delta} (\vec{J} \times \vec{B}) ds = \mu_0 \hat{s} \int_{s=a}^{a+\delta} J(s) \int_{s'=s}^{a+\delta} J(s') ds' ds$$

CONSIDER: $\frac{d}{ds} \int_{s'=s}^{a+\delta} J(s') ds' = -J(s)$ (DISCUSS)

$$\frac{d}{ds} \left[\int_{s'=s}^{a+\delta} J(s') ds' \right]^2 = -2J(s) \int_{s'=s}^{a+\delta} J(s') ds'$$

SO: $\int J(s) \int_{s'=s}^{a+\delta} J(s') ds' ds = -\frac{1}{2} \left[\int_{s'=s}^{a+\delta} J(s') ds' \right]^2$ *

$$\therefore \vec{P} = \frac{-\mu_0 \hat{s}}{2} \left[\int_{s'=s}^{a+\delta} J(s') ds' \right]^2 \Big|_{s=a}^{a+\delta}$$

$$= \frac{-\mu_0 \hat{s}}{2} [0^2 - K_0^2] = \frac{1}{2} \mu_0 K_0^2 \hat{s}$$

$$\Rightarrow P = \frac{B_0^2}{2\mu_0}$$

IN § 7.2.4, GRIFFITHS DERIVES THE ENERGY DENSITY OF THE MAGNETIC FIELD, $B^2/2\mu_0$. COMPARE $\epsilon_0 E^2/2$, EQ. 2.45!

* WHAT WE DID HERE IS REALLY JUST INTEGRATION BY PARTS,

EXAMPLE: GOLD-HOYLE FLUX TUBE

(GOLD, T., & HOYLE, F. 1960, MNRAS, 120, 89)

$$\vec{B} = B_0 \frac{(\hat{z} + ks\hat{\phi})}{1+k^2s^2}$$

- SKETCH THE FIELD.
- FIND \vec{J}
- CALCULATE $\vec{J} \times \vec{B}$

$$\begin{aligned} \mu_0 \vec{J} &= \nabla \times \vec{B} = -\hat{\phi} \frac{\partial B_z}{\partial s} + \frac{\hat{z}}{s} \frac{\partial}{\partial s} (s B_\phi) \\ &= \frac{B_0 \hat{z}}{s} \frac{\partial}{\partial s} \left(\frac{ks^2}{1+k^2s^2} \right) - B_0 \hat{\phi} \frac{\partial}{\partial s} \left(\frac{1}{1+k^2s^2} \right) \\ &= \frac{B_0 \hat{z}}{s} \left(\frac{2ks}{1+k^2s^2} - \frac{2k^3s^3}{(1+k^2s^2)^2} \right) - B_0 \hat{\phi} \left(\frac{-2k^2s}{(1+k^2s^2)^2} \right) \\ &= B_0 \hat{z} \left(\frac{2k(1+k^2s^2) - 2k^3s^2}{(1+k^2s^2)^2} \right) + B_0 \hat{\phi} \frac{2k^2s}{(1+k^2s^2)^2} \end{aligned}$$

$$\mu_0 \vec{J} = B_0 \frac{(2k\hat{z} + 2k^2s\hat{\phi})}{(1+k^2s^2)^2} \quad \star$$

$$\vec{J} = \frac{2k}{1+k^2s^2} \vec{B} / \mu_0 \quad !$$

→ WHAT IS $\vec{J} \times \vec{B}$? DISCUSS.

→ IF YOU WERE GIVEN \star , HOW WOULD YOU CALCULATE \vec{B} ? DISCUSS BOTH B_z AND B_ϕ . (WHY NOT B_s ?)