An Analytic Model of an Isothermal Solar Wind Plasma Interacting with a Neutral Interstellar Component

M Weber

December 19, 1997

Abstract

The Sun's heliosphere is essentially a volume of hot, diffuse plasma, which displays many interesting and complex aspects of astrophysics. The study of this medium has increased our knowledge of more remote areas of the universe, and vice versa. Much of the research currently being done at the forefront of this field involves sophisticated numerical methods, and there has been a concurrent evolution of the models which provide the context for our physical understanding. This paper is an effort to model the charged and neutral components of the heliospheric medium, in an analytic way, to determine how much can be qualitatively deduced about the solar wind from general principles. Two fluids are used to describe an isothermally expanding, hot solar wind, and a cool inflow of neutral hydrogen from the interstellar medium. They are allowed to interact via electronic charge transfer, and the fluid profiles of the two media and of the reaction products are determined under straightforward arguments of self-consistency and qualitative consistency with the actual, observed system. Although this system does not take the existence of a solar magnetic field into account, it could be adapted to one in a physically self-consistent way.

1 Introduction

The thermal conditions of the solar corona are such that the Sun's gravity is insufficient to confine the plasma to a static, finite volume. The hot, ionized gas expands radially to fill space to an arbitrary distance. Furthermore, the constraint of gravity serves to accelerate the outflow to supersonic speeds, in a manner analogous to that of a De Laval jet nozzle [Brandt, 1970]. The high temperatures (order 1 MK) ensure that the gases are almost completely ionized, so the equations of magnetohydrodynamics must be used instead of the simpler set for hydrodynamics plus thermodynamics.

The solar wind evolves in a non-linear, dynamic way, and has non-trivial structure which can be traced back to the magnetic organization of the low ($r \leq 5R_{\odot}$) corona. In particular, there are three categories of solar wind which have been statistically well-determined (inside of 1 AU) (see, e.g., Withbroe et al., 1991; and references therein): high-speed, slow-speed, and transient winds. Transient flows are associated with solar flares and eruptions. The sources of the high-speed wind are commonly considered (cf. Bai et al. [1997]) to be coronal holes and features of similar magnetic configuration, i.e. regions of "open" unipolar field extending to virtual infinity. In comparison, the location and physical nature of the origins of the slow-speed winds are ambiguous. However, Gosling et al. [1981] and others have accumulated much evidence that points to helmet streamers and their interfaces with open field regions as the surface areas corresponding to these mass flows.

The importance of the solar magnetic field to the ordering of the corona and winds is a consequence of the physical parameters of the low-coronal plasma. This layer of the solar atmosphere is a low β_p plasma, meaning that the magnetic pressure term dominates the gas pressure term of the force equation. Within this regime, it is possible for the magnetic field to confine large amounts of coronal plasma, as with helmet streamers, for example. The ratio β_p increases with height, and at some altitude the plasma becomes thermally driven. The subsequent expansion carries any local magnetic flux with it. Hence, there is a non-zero magnetic field throughout the heliosphere, which travels with the bulk motion. Although the field is directly inconsequential to the bulk dynamics of the solar wind, it is indirectly significant via non-linear effects and the introduction of spatial anisotropy.

In a circular fashion, the magnetic field may also play a role in the creation of the solar wind. Radiative efficiency of a plasma depends upon the density. The nebulous corona retains its high temperatures well against radiative losses, but conversely is difficult to heat. Furthermore, it is clear that a thermal pressure gradient is insufficient as a sole mechanism (viz. adiabatic expansion) for producing a supersonic expansion as described above [Withbroe et al., 1991]. The mechanism(s) of coronal heating remains unidentified (although see Ref [10]). It is likely that some combination of magnetic waves and wave-particle interactions are involved (see reviews by Holzer, 1988; and Leer, 1988), but alternative mechanisms (or modifications) have been considered, such as heavy ion beams.

In reality, the solar wind does not expand to infinity. The Sun exists within an interstellar medium, and the solar wind dominates that plasma only to a point such that their total pressures are approximately equal. This volume defines the heliosphere. The solar magnetic field acts to shield the inner solar system from interstellar charged particles, but neutral particles are free to penetrate to a distance determined by ionization rates, at which point the new ions propogate back out of the heliosphere along magnetic field lines. The ionization rates, in turn, are determined mostly by solar radiation (for helium) and charge transfer processes (for hydrogen). The plasma is essentially collisionless throughout the outer heliosphere, and so collisional ionization is relatively negligible there. The ionized interstellar species then contribute to the wind's total distribution function. Their own distribution may or may not equilibrate with the solar ions over the advection timescale [Isenberg, 1986].

This paper attempts to analytically model the heliospheric medium by making several physically reasonable simplifications. We begin with a solar wind velocity profile consistent with Parker's solution [1958] for a single, isothermal fluid. This velocity profile, plus the isothermal condition, plus the ideal MHD equations form the basis of the model. The composition of the wind is considered to be ionized hydrogen. The electrons are presumed (for the sake of charge neutrality) to share the number density and bulk velocity of the protons. There is no magnetic field. An interstellar medium of neutral hydrogen is posited at virtual infinity (which corresponds to the heliospheric terminal shock), with an initially inward bulk flow. This arrangement is comparable to considering the "upstream" direction for a full three-dimensional model wherein the Sun is moving relative to the local interstellar medium (LISM). Infalling hydrogen can be ionized by charge transfer with a solar wind proton. Electrons are presumed to be immediately assimilated to the solar wind. This presumption is also made for the produced, or "pickup", protons, under the argument for the existence of a two-stream instability on timescales much shorter than any other relevant physical process. It is shown that the "pickup" proton fluid is reasonably consistent with the solar proton solution. Measured values are used wherever possible for physical parameters and considerations.

2 First Fluid: The Protonic Solar Wind

The fundamental relations of the model are collected in this section, and are utilized to describe the dominant plasma of the heliosphere. These are (a) the ideal MHD equations; (b) the isothermal condition $T(r) = T_o$ ($\gamma = 1$); and (c) a physically reasonable velocity profile $w(r) > 0$ to describe a supersonic, expanding fluid. The system is spherically symmetric ($\partial_\theta = \partial_\phi = 0$) and steady-state ($\partial_t = 0$), with zero magnetic and electric fields.

The continuity, force, and isothermal relations may be written as

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \rho w) = 0, \tag{1}$$

$$w\frac{dw}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \vec{\nabla}\Phi_g \cdot \hat{\mathbf{r}}, \tag{2}$$

$$p = nk_B T_o = \rho\frac{k_B T_o}{m}, \tag{3}$$

where n, ρ, w, p, T_o, m, and k_B are the number density, mass density, solar wind speed, pressure, temperature, particle mass, and Boltzmann's constant, respectively. It will turn out to be more convenient to work in terms of number density rather than mass density. The gravitational potential Φ_g is

$$\Phi_g = -\frac{GM_\odot}{r},$$

where G is the gravitational constant, and $M_\odot$ is the mass of the Sun. Equations (1) and (3) can be used to solve for n and p, and to eliminate these quantities explicitly from the force equation. This leaves

$$\rho(R) = \frac{C_1}{R^2 W}, \tag{4}$$

$$\frac{d}{dR}\left[\frac{1}{2}W^2 - \ln W - 2\ln R - \frac{\Psi_g}{R}\right] = 0, \tag{5}$$

$$p(R) = \rho(R) v_T^2, \tag{6}$$

where R and W are the dimensionless coordinate and velocity, respectively:

$$R \equiv \frac{r}{a},$$

$$W \equiv \frac{w}{v_T}.$$

Also introduced here are the characteristic quantities for radius, density, thermal velocity, and gravitational velocity.

$$a \equiv R_\odot,$$
$$C_1 \equiv \frac{|M_\odot|}{4\pi a^2 v_T},$$
$$v_T^2 \equiv \frac{k_b T_o}{m},$$
$$v_g^2 \equiv \frac{GM_\odot}{a}.$$

The ratio Ψ_g is such that

$$\Psi_g \equiv \frac{v_g^2}{v_T^2} \tag{7}$$

represents a dimensionless gravitational potential energy. Equation (4) expresses conservation of mass (number), so C_1 represents the solar wind mass source, i.e. the mass loss rate of the Sun.

The remaining step is to solve equation (5) for $W(R)$. This equation is transcendental, and is best solved numerically. Figure 1 displays some solution curves. Solutions #1 and #2 are the only physically reasonable solutions for the solar outflow, and only #1 becomes supersonic.

This classic result for a supersonically expanding wind is due to Parker [1958]. Its simple elegance lies in its qualitative capture of a reasonably correct solution. In this paper, an analytic "semi-Parker" solution is used for the velocity $W(R)$.

Equation (5) can be written as

$$f(W) + g(R) = C_2, \tag{8}$$

where

$$f(W) = \frac{1}{2}W^2 - \ln W,$$
$$g(R) = -2\ln R - \frac{\Psi_g}{R}.$$

Each of these functions has a single extremum, the ordered pair of which corresponds to the saddle point of Figure 1 (and hence lies on the supersonic solution). This point $(R_{cr},\, W(R_{cr}))$ is

$$R_{cr} = \frac{\Psi_g}{2}, \tag{9}$$

$$W(R_{cr}) = \frac{w(R_{cr})}{v_T} = 1. \tag{10}$$

Thus the critical radius R_{cr} is where the wind becomes supersonic.

If one considers the limit where R, $W \to 0$, equation (8) can be approximately solved as

$$W(R) = \exp\left(-C_2 - \frac{\Psi_g}{R}\right), \tag{11}$$

$$\lim_{R\to\infty} W(R) = \exp\left(-C_2\right) \equiv W_\infty.$$

Technically speaking, the integration constant C_2, and hence the flow at infinity, is uniquely determined by equation (8) at the critical point, but the approximate solution for small R, W has some problems there. Nevertheless, the *form* of equation (11) is qualitatively close to the exact solution. A two-parameter solution for the velocity can be written down, where the parameters are fixed by (a) the critical point; and (b) the velocity at infinity W_∞.

$$W(R) = \exp\left[\alpha\left(1 - \frac{R_{cr}}{R}\right)\right], \tag{12}$$

$$\alpha \equiv \ln W_\infty,$$

$$R_{cr} \equiv \frac{\Psi_g}{2}.$$

The fundamental relations are equations (4), (6), and (12), with the associated definitions relative to the physical parameters. These can now be used to describe the protonic solar wind. This one-fluid solution can then be used as a context for introducing more physics to the model (e.g. an influx of interstellar neutral hydrogen), subject to consistency considerations.

The single-fluid solution (equations (4), (6), and (12)) may now be applied to protons of solar origin. The model includes electrons, but their solution ($n_e(R)$, $W_e(R)$) is identified with the protons' solution ($n_p(R)$, $W_p(R)$), in order to preserve charge density neutrality of the solar wind and Sun. Due to this ad hoc treatment of the solar electrons, and because of their relatively negligible mass, they are discounted from directly affecting the wind solution.

Table 1 lists the values used for the various physical parameters of the model, and for their calculated derivations. The velocity parameter α was, in fact, chosen to fit $w(1AU)$, instead of w_∞, to observational data [Feldman et al., 1977]. The temperature T_o is the order of magnitude of coronal temperatures. The heliospheric terminal shock (TS) is set (as the model's virtual infinity) at 100 AU.

In comparison with observed values (for $r \leq 1AU$), the predicted number densities $n_p(R)$ (see Figure 2) are high by about a factor of 10. The predicted velocity $w_p(R)$ (see Figure 3) drops to zero too quickly near the Sun. However, the number flux density $n_p w$ (determined by the value used for the solar mass loss rate) agrees to one significant figure. These results suggest that this simplistic model of the solar wind might be qualitatively acceptable at a variation of one order of magnitude.

A comparison of relevant timescales can be made (Figure 4), using the isothermal temperature T_o and the following formulae:

$$\tau_{C,e} = 3.44 \times 10^5 \, \frac{T_{eV}^{\frac{3}{2}}}{n_e \lambda_C} \text{ sec},$$

$$\tau_{C,p} = 2.09 \times 10^7 \, \frac{T_{eV}^{\frac{3}{2}}}{n_p \lambda_C} \text{ sec},$$

$$\tau_W = \frac{r}{w} = 7648 \, \frac{R}{W} \text{ sec},$$

where $\tau_{C,e}$ and $\tau_{C,p}$ are the Coulomb collision times for electrons and protons, respectively, $\lambda_C \simeq 20$ is the Colulomb logarithm, and τ_W is the characteristic time of the wind flow. It is self-evident that Coulombic relaxation of the electron and proton plasmas will be significant inside the critical point R_{cr}. The low solar corona is an excellent conductor, so it is a reasonable assumption that both species will have Maxwellian distributions, and be in thermal equilibrium, for subsonic wind speeds. As the wind accelerates towards, and through the critical point, the two species become effectively collisionless, the protons being first. From this point onwards, the two species are considered to not be interacting or collisionally evolving. It would be appropriate to apply the adiabatic condition to the subsequent development of the distributions, but this model posits an unknown heating mechanism to maintain isothermality. As the Maxwellian distribution is parameterized solely by the thermal velocity (which is in turn a constant under isothermality), the isothermal condition maintains the Maxwellian distributions of the two species. The heating mechanism acts to balance the plasma's tendency to adiabatically cool. A physical candidate for a heating mechanism would be hydromagnetic waves interacting with plasma particles, but this model does not take into account a magnetic field.

In this section a "semi-Parker" model of the protonic (and, by association, electronic) plasma has been shown to support a solar wind which is collisional at subsonic speeds, where it enjoys Maxwellian distributions and thermal equilibrium. The plasma quickly becomes collisionless near the sonic point. The evolution of the distribution can only happen adiabatically, but the isothermal condition requires the assumption of a heating mechanism which prevents this, so the distribution is fixed. Given reasonable values for the physical parameters, the model returns values for the plasma which disagree with observational data by approximately an order of magnitude.

3 Second Fluid: Interstellar Hydrogen

The outward ram pressure of the solar wind is $\rho|\mathbf{ww}| = \rho w_\infty^2$, and drops as r^{-2} at large distances. (The thermal pressure p is $\rho v_{T,p}^2 \ll \rho w_\infty^2$.) At some point this pressure becomes comparable to the pressure of the LISM. This condition marks the extent of the heliosphere. For the case wherein the solar system has some velocity with respect to the LISM, the fluid fluxes of both media must turn at the heliopause and follow its surface in a downstream direction. (It is not possible for the supersonic wind to do this— hydrodynamics dictates that the wind forms a shock front (the TS) to decelerate to subsonic speeds before encountering the heliopause.) Charged populations of the LISM are repelled from entering the heliosphere by the solar magnetic field and fluid-flow. Neutral particles may freely pass into the solar system due to the relevant number densities being so low. However, there are various processes which can act to ionize such particles, at which time they are shunted back out of the system along the magnetic field lines. The two most important ionization mechanisms are photoionization (of neutral helium) and charge transfer (for neutral hydrogen).

In this section a second fluid is introduced to the model, to incorporate an influx of neutral hydrogen to the heliosphere from the LISM. The particles originate at virtual infinity with some initial density $n_H = n_{H\infty}$ and inward radial velocity $u_H = -|u_{H\infty}|$. Table 1 lists observational estimates of LISM quantities. We see that the temperature $T_{\mathrm{LISM}} \ll T_o$, so we approximate this species as a cold gas, viz.

$$f_H(R, \vec{\mathrm{v}}) = n_H(R)\delta(\vec{\mathrm{v}} - \vec{\mathbf{u}}_{\mathbf{H}}(R)). \tag{13}$$

The velocity $U_H(R)$ is determined by gravitational infall,

$$U_H(R) = -\left[|U_{H\infty}|^2 + 4\frac{R_{cr}}{R}\right]^{\frac{1}{2}}, \tag{14}$$

$$U_H(R) \equiv \frac{u_H(R)}{v_T},$$

where the referential thermal velocity v_T is that of the *solar wind protons*.

The two fluids interact via an idealization of the charge transfer process, wherein a solar wind proton takes on the electron of an interstellar hydrogen. The (new) solar hydrogen (species H') will have the temperature and bulk speed of the solar wind. The (new) interstellar proton (species p') will have zero temperature, and the bulk speed of the interstellar hydrogen. This process reduces the number of interstellar neutrals by one. The full form of the continuity equation must be used, which incorporates this process, to derive the density function $n_H(R)$.

$$\vec{\nabla} \cdot (n_H \vec{\mathbf{u}}_{\mathbf{H}}) = \frac{dn_H}{dt}. \tag{15}$$

The loss rate of interstellar hydrogen can be found in a straightforward manner. A particle μ, of constant interaction cross-section σ, will sweep out a volume $\sigma|\vec{\mathbf{v}}_\mu|\Delta t$ of the medium ν in time Δt. For $\vec{\mathbf{v}}_\nu||\vec{\mathbf{v}}_\mu$, the probability $\mathcal{P}_1$ of a collision in time Δt is equal to the number N_ν of particles which pass through this volume in time Δt, so

$$\begin{aligned} \mathcal{P}_1 &= n_p\sigma(\Delta r + w\Delta t), \\ &= n_p\sigma(\frac{\Delta r}{\Delta t} + w)\Delta t, \\ &= n_p\sigma(w - u)\Delta t, \end{aligned} \tag{16}$$

and thus the decay rate of species μ is

$$\frac{dn_H}{dt} = \sigma n_p n_H(u - w), \tag{17}$$

under the assumption that the collision eliminates the particle μ.

The expression (17) can be used with the continuity equation (15) to solve analytically for $n_H(R)$.

$$n_H(R) = \frac{C_3}{R^2|U_H(R)|}\exp\left\{f\left[\frac{1}{\alpha}\left(\frac{1}{W_{p\infty}} - \frac{1}{W(R)}\right) + \frac{1}{2}\left(U_H(R) - U_{H\infty}\right)\right]\right\}, \tag{18}$$

or

$$n_H(R) = \frac{C_3}{R^2|U_H(R)|}\exp\left\{\frac{F}{W(R)} + GU_H(R) + H\right\}, \tag{19}$$

where C_3 is a constant of integration and

$$f \equiv \frac{a\sigma C_1}{m_p R_{cr}},$$

$$F \equiv -\frac{f}{\alpha},$$

$$G \equiv \frac{f}{2},$$

$$H \equiv f\left(\frac{1}{\alpha W_{p\infty}} + \frac{|U_{H\infty}|}{2}\right).$$

The charge transfer cross-section σ_{ct} is a relatively constant function of temperature and can be well-approximated for the purposes of this model as

$$\sigma = \sigma_{ct} = 3 \times 10^{-15}\ \mathrm{cm}^2,$$

[Tawara et al., 1983]. Note that this is about two orders of magnitude larger than the Bohr cross-section σ_B, which would be appropriate for kinetic collisions and is also a constant function of space and temperature. Throughout the supersonic region, the relative kinetic energy of the two fluids is greater than or equal to the thermal energy $k_B T_o \simeq 85$ eV $\gg 13.6$ eV $=$ ionization energy of hydrogen. Thus, it would be reasonable to make the approximation of $\Delta N_H = -1$ for every kinetic collision, and adjust the total cross-section σ at the one percent level. The form of the solution of the density $n_H(R)$ would not change. However, the ramifications for the products are different for charge transfer and kinetic ionization, so the model is simplified by only including the former process. This choice of σ equal to σ_{ct} determines the following values:

$$f = 23,$$

$$F = -13,$$

$$G = 11,$$

$$H = 5.2.$$

The constant C_3 is set by the density of the interstellar neutrals at model infinity, i.e. just within the TS which has been located at $r = 100$ AU. The density n_{LISM} of the "local fluff" surrounding the solar system is approximately .1 cm^{-3}. Some of the hydrogen atoms undergo charge exchange processes outside the heliopause, and are diverted into the ionized plasma following the heliosheath. More particles are ionized in the region of subsonic solar wind, just upstream of the TS. Calculations suggest that the neutral hydrogen density is reduced in these regions by approximate factors of 100 and 20, respectively [Holzer, 1989]. Correspondingly,

$$C_3 = 5.8 \times 10^3 \mathrm{cm}^{-3},$$

so that

$$n_{H\infty} \equiv n_H(100AU) = 5 \times 10^{-5} \mathrm{cm}^{-3}.$$

Equations (14) and (19) describe the influx of interstellar neutral hydrogen as a fluid. The distribution function is given by equation (13). Particles are permanently removed by charge transfer with the solar protons. Photoionization and kinetic ionization are not included.

4 Third Fluid: The Hydrogenic Solar Wind

One of the products of the charge transfer process described in the preceeding section is a neutral hydrogen atom from a solar wind proton. At the point of origin, this population will have the velocity and temperature profile of the solar wind. It is presumed that neutral particles are not subject to the isothermal heating mechanism affecting the charged solar wind, so the distribution is expected to evolve. As this wind continues moving outwards, it will cool adiabatically while being replenished with hotter (at the isothermal temperature) particles. The solution to the distribution and fluid variables for given R involves a spatial integral from R to the Sun over a source term of number density and thermal energy which is a function of radius.

Some straightforward considerations will provide a simpler solution. Figure 5 is a plot of $\Delta N_{H'}$ ($= \Delta N_{ct} = -\Delta N_H$) as a function of R. The maximum generation of solar hydrogen takes place in the vicinity of $R = 550$. The area of the curve for $R \geq 216$ is 98.5 percent of the total. In other words, only 1.5 percent of the solar hydrogen is generated within 1 AU.

Referring to Figure 3 for the solar wind velocity profile, we find that $W_p(1AU)$ is already $\simeq .9W_{p,\infty}$ at this distance. The ratio of gravitational potential energy (equation (7)) to the (bulk flow) kinetic energy of a solar proton is

$$\frac{\mathrm{PE_{grav}}}{\mathrm{KE_{bulk}}} = \frac{\Psi_g}{R} \frac{1}{W_P^2}.$$

Most of the solar hydrogen is created at larger radii, with (slightly) higher initial bulk flow and less negative gravitational potential energy. Considered ballistically, most of the neutral particles will escape the heliosphere with virtually no loss of velocity. (Although internal energy will be lost to adiabatic cooling of the gas, there is still sufficient total energy for the supersonic fluid to escape to the TS.)

The approximation for this fluid is twofold. Only one-and-a-half percent of the solar hydrogen is created within the Earth's orbit (by the charge transfer interaction of interstellar hydrogen and solar protons). Hence, the solution begins at $r = 1AU$ ($R = 216$). At greater distances, the velocity is relatively constant and close to the velocity $W_{p,\infty}$ at model infinity. The fluid velocity is set as

$$W_{H'}(R) = 0.9 W_{p\infty}, \tag{20}$$

since (a) this is the location of greatest production, and (b) this value is a reasonable lower limit on the bulk velocity at any greater distance.

The form of the solution for the density follows upon the application of the continuity principle. The density n_H of the interstellar hydrogen goes to zero before reaching the Sun, and is the only source for the solar hydrogen population, so the total amount of hydrogen within the system is conserved. Thus, the density flux $n_H U_H$ of interstellar hydrogen through a surface $S(R)$ must be equal to the density flux of the emerging solar hydrogen $n_{H'} W_{H'}$. This can be seen directly from the continuity equation

$$\begin{aligned} \vec{\nabla} \cdot (n_{H'} \vec{\mathbf{w}}_{\mathbf{H'}}) &= \frac{dn_{H'}}{dt}, \\ &= -\frac{dn_H}{dt}, \\ &= -\vec{\nabla} \cdot (n_H \vec{\mathbf{u}}_{\mathbf{H}}). \end{aligned}$$

The solar hydrogen density is thus

$$n_{H'}(R) = -\frac{U_H(R)}{W_{H'}(R)} n_H(R). \tag{21}$$

Figure 2 includes the density curve for solar hydrogen, which is seen to be small in comparison with the other model densities. The distinction in the densities of the interstellar and solar hydrogen is due to the difference in the magnitudes of their bulk flow. The solar wind moves supersonically, the interstellar particles do not.

It is appropriate to address the issue of interactions for the neutral solar wind. An equation for the timescale of a particle μ interacting with a medium ν can be formed from equation (16), where $\mathcal{P}_1$ is set equal to one.

$$\tau_{x,\mu\nu} = (n_\nu \sigma_\times |\vec{v}_\mu - \vec{v}_\nu|)^{-1}. \tag{22}$$

For head-on collisions with the infalling interstellar hydrogen, the cross-section is four times the Bohr cross-section

$$\sigma_\times = 4\sigma_B = 3.5190 \times 10^{-16} \text{ cm}^2,$$

so

$$\tau_{B,H'H}(R) = (4n_H(R)\sigma_B(w_{H'}(R) - u_H(R)))^{-1}.$$

For charge-transfer interactions with the protonic solar wind,

$$\tau_{ct,H'p}(R) = (n_p(R)\sigma_{ct}(w_p(R) - w_{H'}(R)))^{-1}.$$

The timescale of the flow rate

$$\tau_{W,H'} = \frac{r}{w_{H'}} = \frac{a}{.9 v_T W_{p\infty}} R,$$

is close to that of the protonic wind, and the curves are not distinguishable for the scaling of Figure 4. Comparison of $\tau_{W,H'}$ with $\tau_{H,H'}$ and $\tau_{p,H'}$ indicates that it is reasonable to neglect these interactions for the hydrogenic solar wind.

Equations (20) and (21) are the solutions for the hydrogenic solar wind, within this model. This population is generated throughout most of the heliosphere by charge transfer processes between the infalling interstellar medium and the protons of the solar wind. The gas cools adiabatically as it moves out of the solar system, but particles at the isothermal wind temperature are continually introduced. The distribution function is thus somewhat complicated, and no attempt is made to solve it here. The fluid does not undergo any significant interactions with the other fluids of the model.

5 Fourth Fluid: The "Pickup" Protons

The final population addressed in this model belongs to the pickup, or interstellar, protons. These are the remains of the infalling hydrogen atoms as they are stripped of their electrons by the solar protons, via the charge transfer mechanism. As the proton carries the majority of the mass of a hydrogen atom, the pickup protons are left with an initial velocity equal to that of the local hydrogen influx, and are produced at the same rate as the solar hydrogen particles. Technically, a solution of the distribution function $f_{p'}(R, \vec{v})$ would involve an integration of the source function from R to infinity. An extra consideration, which was not necessary for the case of the solar hydrogen, is the influence of the isothermal-maintenance mechanism, which is presumed to affect charged particles. Again, the straightforward application of some physics, within the constraints of the model, allows a simple and direct solution.

The charge transfer mechanism produces a net current. From the point of interaction, an extra electron moves out with the solar wind, and an extra proton falls inward with the interstellar hydrogen, resulting in a net current density $\vec{j}$ directed inward. This situation would violate two explicit constraints of the model: (a) that the Sun and solar wind are neutrally charged at all time t; and (b) that there are no electric or magnetic fields (by Ohm's Law).

In the frame of the solar wind, a pickup proton appears with a net velocity

$$u_{p'}(R) = u_H(R) - w_p(R) < 0,$$

which is several times the thermal velocity. This situation is subject to a "two-stream" instability for inhomogeneities on a length scale L_{2S} such that

$$L_{2S} \geq \frac{1}{k_{cr}},$$

where the critical wavenumber k_{cr} is

$$k_{cr} = \frac{\omega_{pl,e}}{|u_{p'}|} \left[1 + \frac{3}{2} \left(\frac{\omega_{pl,p'}}{\omega_{pl,e}} \right)^{\frac{2}{3}} \right], \tag{23}$$

$$\omega_{pl,\mu} \equiv \left(\frac{4\pi e^2 n_\mu}{m_\mu} \right)^{\frac{1}{2}},$$

for the solar electron plasma (equation 8.4.4 of Sturrock [1994]). If we take the Debye length λ_D of the solar plasma as the upper limit on spatial fluctuations, we find that

$$\lambda_D k_{cr} \simeq 7 \times 10^7 \gg 1,$$

which shows that it is possible for the "graininess" of the plasma itself to produce unstable wave modes. The characteristic timescale for the maximum growth rate of a two-stream instability is

$$\tau_{2S} = \frac{2}{\omega_{pl,e}},$$

which is less than 1 millisecond throughout the heliosphere. This number is instantaneous compared to any other timescale in the model (see Figure 4). The ad hoc treatment which this model gives to this situation is to claim that the plasma is continually adjusting (in a steady-state manner) by instantaneously (viz., on the timescale of the instability growth) acclimitizing the pickup protons to the solar wind. The isothermal mechanism acts instantaneously to bring the pickup ions to thermal equilibrium with the ambient plasma, as well. As a result, the "current density problem" is solved, as both of the "extra", and oppositely charged, particles now move outward together at the solar wind speed.

By this arrangement, the solutions for the density and velocity of the pickup protons (and the "extra" solar electrons) are trivial. The velocity profile is identical to that of the solar wind (equation 11).

$$U_{p'}(R) = W_p(R). \tag{24}$$

The number density of the pickup protons is determined by the same application of the continuity principle that was used for the solar hydrogen (equation 21). So the relation for the number density is

$$n_{p'}(R) = -\frac{U_H(R)}{U_{p'}(R)} n_H(R). \tag{25}$$

The model is closed by considering the pickup protons (and associated electrons) to be a perturbation to the solar wind plasma, which can be absorbed without effect at the precision of this model.

6 Summary and Discussion

This paper outlines a three-fluid model of the particle flows within the heliosphere. The system is described as spherically-symmetric and steady-state, with no electric or magnetic fields. The solar wind is treated as a plasma of ionized hydrogen which thermally expands. An unidentified heating mechanism is allowed for, which operates uniformly throughout the heliosphere and maintains charged particles at a uniform temperature T_o. This isothermal condition, coupled with the inward force of gravity, allows a supersonic solution for the plasma bulk velocity [Parker, 1958]. An analytic, "semi-Parker" solution for the wind speed is chosen (equation (12)), which provides reasonable (i.e. within an an order of magnitude or so) results in comparison with observational data. The number density function (equation (4)) follows from the continuity equation. The functions are normalized to agree with observed values of (a) the solar wind velocity at 1 AU; and (b) the mass loss rate of the Sun.

The spatial boundary of the model is chosen to lie just within a terminal shock, which would supposedly be produced by a supersonic wind expanding to meet an ambient medium (i.e. the LISM). A second fluid is included in the model to describe an inflow of neutral hydrogen through this boundary with some non-zero initial velocity. (In a fully three-dimensional model, this would correspond to the description of a Sun with a velocity relative to the interstellar medium, in the "upstream" direction.) The velocity solution (equation (14)) is simply that for gravitational infall. The temperature of the interstellar medium is presumed to be sufficiently cold that thermal attributes are negligible, and that the distribution function is essentially a delta function. Interaction of the interstellar hydrogen with the solar plasma is allowed via charge transfer, whereby a solar wind proton accepts the electron of an interstellar hydrogen atom. This produces a "solar" hydrogen atom, moving initially at the wind speed, and an "interstellar" proton, moving initially inward at the speed of the neutral influx. This mechanism gives a "loss" term which must be included in the continuity equation for the number density of the interstellar hydrogen. The solution of this equation gives a relation (19) for this quantity. Virtually all of the interstellar hydrogen has been lost at $r \simeq .5AU$.

The velocity of each solar hydrogen atom is initially that of the local solar wind. Since the isothermal heating mechanism is assumed to not affect neutral particles, this population must cool adiabatically as it expands outward. A significant fraction of these particles are produced above a sufficiently large radius, and with a large enough total energy, that they can easily escape the system with no significant decline in velocity. Hence, their speed is given as a constant function of radius (20), the value being equal to that of the region of greatest creation and lowest total energy. The density (21) is determined by requiring that the number flux of interstellar hydrogen inwards be equal to the number flux of solar hydrogen outwards. A consideration of timescales for the charge transfer and kinetic interactions, with that of the bulk velocity, for the solar hydrogen indicates that this population is collisionless and leaves the system unperturbed.

The other product of the ionization of an interstellar hydrogen atom is a proton. Two significant mechanisms act upon the particle instantly at creation. The isothermal condition now applies, and a two-stream instability with the outflowing solar plasma develops. As a simple resolution, the proton is treated as being instantly assimilated to, and thermalized with, the charged solar wind. Hence the designation of "pickup protons" is applied as an indication of their origin. Another result of this resolution is that the model constraints of no electromagnetic fields, and charge neutrality, remain inviolate. The velocity profile of the pickup protons (and extra solar wind electrons) (24) automatically agrees with the solar wind, and the density is determined by continuity as the solar hydrogen was (25). Thus, the pickup protons can be treated as a direct addition to the distribution of the solar wind protons. The number density is sufficiently low as to not materially affect that previously determined solution.

Upon creation, a pickup proton has an initial velocity directed oppositely to the solar wind, and of a magnitude several times ($\sim 5\times$) larger than the thermal velocity in the wind frame. The continual resolution of the two-stream plasma instability is presumed to rapidly disperse the pickup proton's momentum and energy to the solar protons. This results in a net increase in the energy of the protonic wind, and a net decrease in its momentum. In other words, the equilibration of the pickup protons to the solar wind produces excess heat and a momentum drag. It is apparent in Figure 5 that the instantaneous amount of new pickup protons is about four and a half orders of magnitude smaller than the ambient solar proton population. The average velocity of the pickup protons mentioned above suggests that the momentum drag is a less than 0.1% effect. Furthermore, the isothermal mechanism works near-instantaneously in this model to enforce a Maxwellian distribution at the isothermal temperature. Physically then, this mechanism absorbs the momentum drag. Hence, the continual introduction of pickup protons does not materially affect the solar wind solution.

In passing, it may also be observed, as for the momentum drag, that the heating provided by the pickup protons is negligible. The amount is approximately four orders of magnitude too small to maintain the isothermal temperature of the solar wind against adiabatic cooling, so "ion beam heating" by the pickup protons is not a viable candidate for the isothermal mechanism.

Based on this model, the following statements about the heliosphere could be made. The number (and mass) density throughout the solar system is dominated by the protonic solar wind. There is a smaller, but not insignificant, amount of interstellar neutral hydrogen which penetrates to approximately the Earth's orbit before it is effectively ionized by charge transfer with the protonic wind. There is a hotter, faster population of hydrogen which flows outwards from the Sun, but which is too tenuous to be ionized via collisions. Plasma waves are being continuously generated in the outer regions of the heliosphere, which convert the kinetic energy of the neutral influx immediately into heat, and which are effective at scattering particles.

This model can provide a basis for further work. Having accounted for two main populations of the heliosphere, a small magnetic field for the Sun can be added. Such a field need not provide enough magnetic pressure to significantly affect the force equation, but would help explain why the solar electrons move at the same bulk velocity as the protons. Also, the redirection of pickup protons could be addressed as sudden affixation to field lines upon ionization, which are in turn continually swept outward by the supersonic wind. This freedom would allow more careful consideration of how well the pickup protons are assimilated to the solar wind [Isenberg, 1986], and by what mechanisms. The magnetic field also allows a natural way to explain the energy deposition imposed in this model via the isothermal condition. Hydromagnetic waves propogating outwards from the Sun may gradually deposit energy to the particles. Finally, a magnetic field provides for a way to introduce charged components to the interstellar composition without precipitating unobserved amounts into the solar system.

A Figures

B Relevant Physical Values for the Heliospheric Model

References

[1] Bai, T., J. T. Hoeksema, M. Weber, and L. W. Acton. Solar Origin of the 26-day Periodicity Observed by *Ulysses*. *J. Geophys. Res.* 102:9793. 1997.

[2] Brandt, J. C. *Introduction to the Solar Wind.* (WH Freeman & Company). 1970.

[3] Feldman, W. C., J. R. Ashbridge, S. J. Bame, and J. T. Gosling. Plasma and Magnetic Fields from the Sun. In *The Solar Output and Its Variations*, ed. O. R. White (Colorado Assoc. Univ. Press, Boulder), p. 351. 1977.

[4] Gosling, J. T., G. Borrini, J. R. Ashbridge, S. J. Bame, W. C. Feldman, and R. T. Hansen. Coronal Streamers in the Solar Wind at 1 AU. *J. Geophys. Res.* 86:5438. 1981.

[5] Holzer, T. E. Acceleration of Stellar Winds. In *Proc. Sixth Intl. Solar Wind Conf.*, eds. V. J. Pizzo, T.E. Holzer, and D. G. Sime, NCAR TN-306 (Natl. Center for Atmos. Res., Boulder), pp. 3–22. 1988.

[6] Holzer, T. E. Interaction Between the Solar Wind and the Interstellar Medium. *Ann. Rev. Astron. Astrophys.* 27:199. 1989.

[7] Huba, J. D. *NRL Plasma Formulary*, 1994 revised, NRL/PU/6790–94-265, p. 37. 1994.

[8] Isenberg, P. A. Interaction of the Solar Wind with Interstellar Neutral Hydrogen: Three-Fluid Model. *J. Geophys. Res.* 91:9965. 1986.

[9] Leer, M. A. The Solar Wind Termination Shock and the Heliosphere Beyond. In *Proc. Sixth Intl. Solar Wind Conf.*, eds. V. J. Pizzo, T.E. Holzer, and D. G. Sime, NCAR TN-306 (Natl. Center for Atmos. Res., Boulder), pp. 3–22. 1988.

[10] Savage, D., and W. Steigerwald. "Solar Mystery Nears Solution with Data from SOHO Spacecraft". NASA Press Release 97-256. November 5, 1997.

[11] Parker, E. N. *Ap. J.* 128:664. 1958.

[12] Sturrock, P. A. *Plasma Physics: An Introduction to the Theory of Astrophysical, Geophysical & Laboratory Plasmas* (Cambridge Univ. Press, Cambridge). 1994.

[13] Tawara, H., T. Kato, and Y. Nakai. *Rep. IPPJ-AM-30.* Inst. of Plasma Phys., Nagoya Univ., Japan. 1983.

[14] Withbroe, G. L., W. C. Feldman, and H. S. Ahluwalia. The Solar Wind and Its Coronal Origins. In *Solar Interior and Atmosphere*, eds. A. N. Cox, W. C. Livingston, and M. S. Matthews, (Univ. of Arizona Press, Tucson), p. 1087. 1991.

Physical Quantity	Symbol	Value
Fundamental Constants		
Boltzmann's constant[a]	k_B	1.3807×10^{-16} erg/deg(K)
Elementary charge[a]	e	4.8032×10^{-10} statcoul
Electron mass[a]	m_e	9.1094×10^{-28} g
Proton mass[a]	m_p	1.6726×10^{-24} g
Gravitational constant[a]	G	6.6726×10^{-8} dyne cm^2/g^2
Bohr atomic cross – section[a]	σ_B	8.7974×10^{-17} cm^2
Empirical Parameters		
Isothermal temperature[a]	T_o	1.00×10^6 deg(K)
Solar radius[a]	a $(R_\odot)$	6.96×10^{10} cm
Solar mass[a]	$M_\odot$	1.99×10^{33} g
Solar mass loss rate[a]	$\dot{M}_\odot$	1.22×10^{12} g/s
Solar wind speed at 1 AU[b]	$W_p(216)$	5.14
LISM temperature[b]	$T_{\rm LISM}$	$\simeq 10^4$ deg(K)
LISM density[b]	$n_{\rm LISM}$	$\simeq 0.1$ cm^{-3}
LISM "drifting" speed[b]	$u_{\rm LISM}$	$\simeq 2.3 \times 10^6$ cm/s
Derived Quantities		
Coulomb logarithm	λ_C	$\simeq 20$
Proton thermal velocity	v_T	9.1×10^6 cm/s
Solar escape velocity	v_g	4.4×10^7 cm/s
Characteristic mass density of solar wind	C_1	2.2×10^{-18} g/cm^3
Characteristic grav.l PE	Ψ_g	23
Solar wind speed at infinity	$W_{p\infty}$	5.66
Transonic radius	R_{cr}	12
Log of $W_p\infty$	α	1.73
LISM density at model boundary	$n_{H\infty}$	$\simeq 5 \times 10^{-5}$ cm^{-3}
LISM speed at boundary	$U_{H\infty}$	0.25
Dimensional factor for $n_H(R)$	C_3	5.8×10^3 cm^{-3}

Table 1: Physical numbers used in this paper. Ref. a is [Huba, 1994]. Ref. b is [Holzer, 1989].